

Sparse Tensor Accelerators: Abstraction and Modeling

Background Lecture Part 2

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Angshuman Parashar

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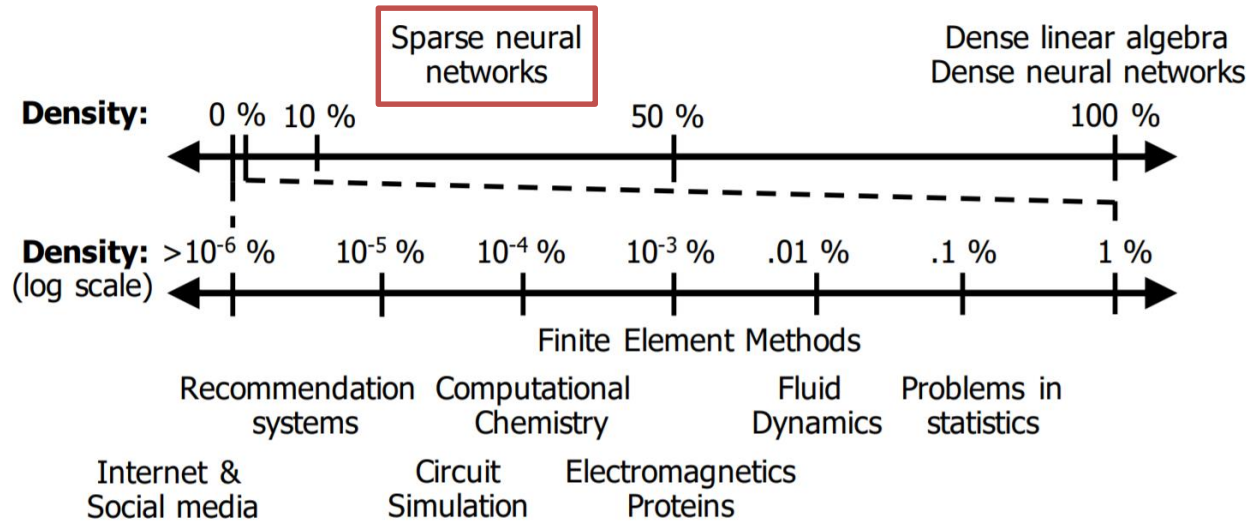
Nellie Wu

ISCA Tutorial

June 2021



Sparse Tensor Algebra in Popular Applications



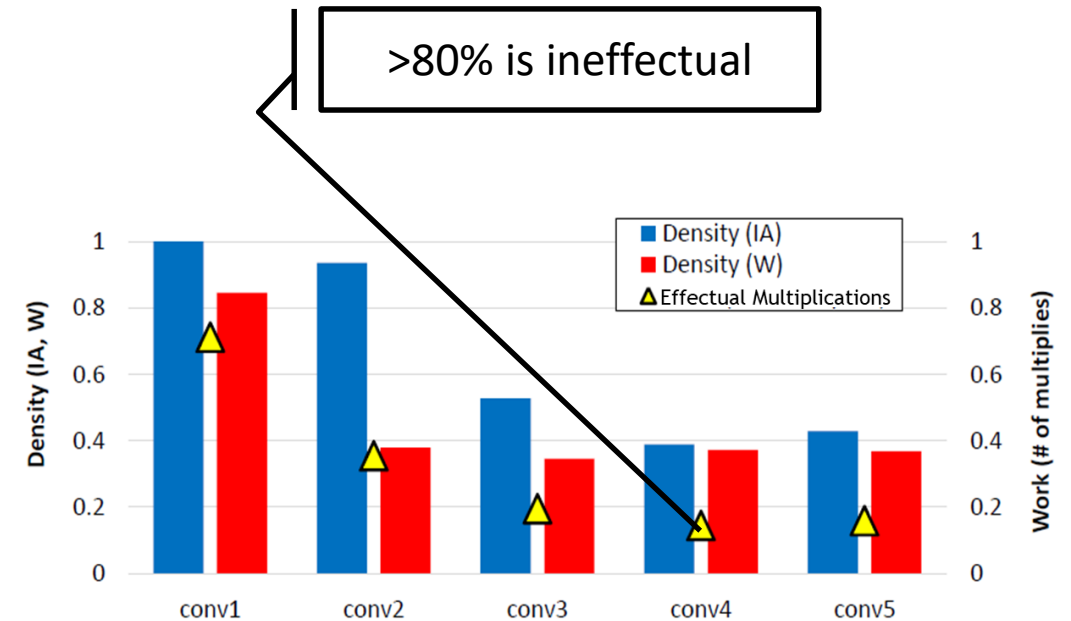
Workload Sparsity by Workload Domain

[Hedge, MICRO19]

0 x Anything = 0

0 + Anything = Anything

Ineffectual Computations



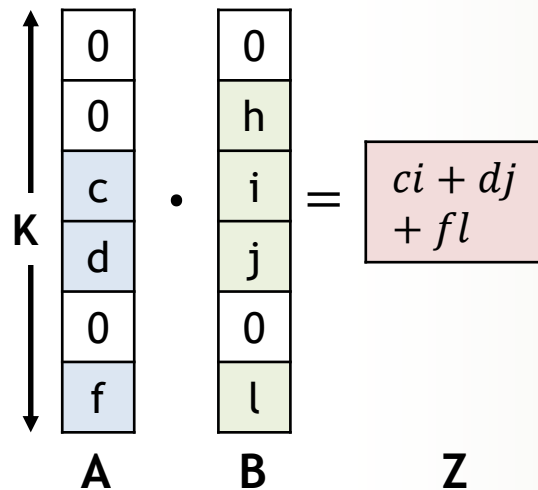
Pruned AlexNet Density

[Parashar, ISCA17]

Processing Uncompressed Sparse Tensor Workloads

Example Workload:
Dot Product of Vectors

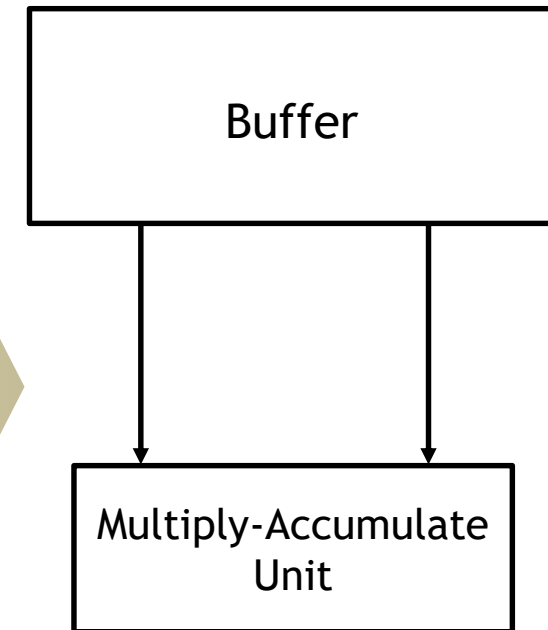
$$Z = \sum_{k=0}^K A[k] * B[k]$$



Mapping
Scheduling of data movement & compute in time & space

```
for k in [0:K)  
  Z += A[k] * B[k]
```

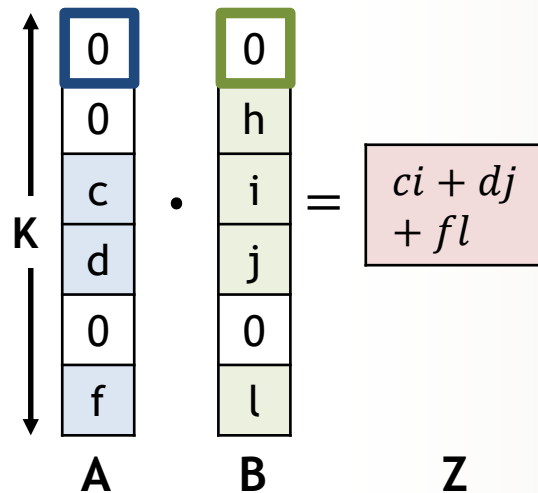
Accelerator Architecture



Processing Uncompressed Sparse Tensor Workloads

Example Workload: Dot Product of Vectors

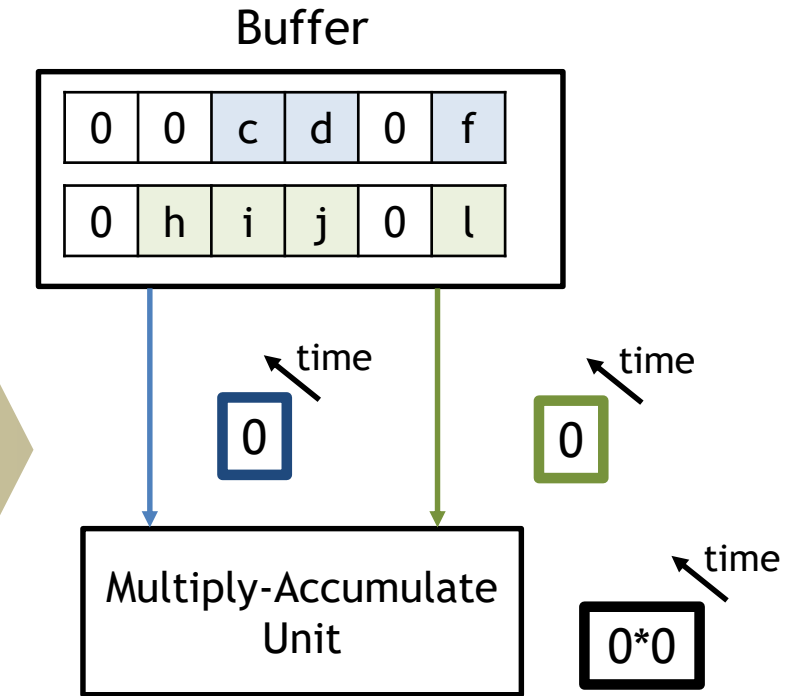
$$Z = \sum_{k=0}^K A[k] * B[k]$$



Mapping
Scheduling of data movement & compute in time & space

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for k in [0:K)  
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Accelerator Architecture

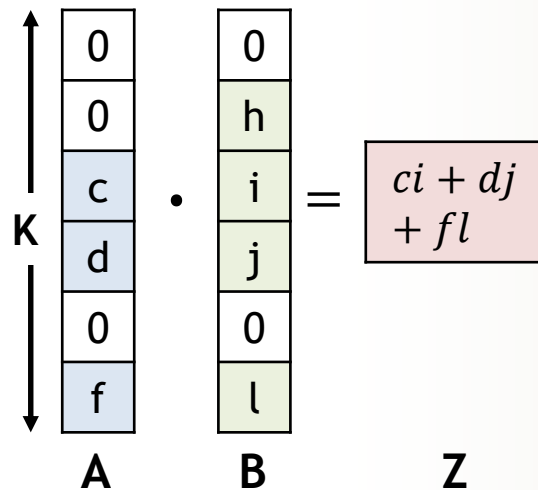


**Z data movements not shown*

Processing Uncompressed Sparse Tensor Workloads

Example Workload:
Dot Product of Vectors

$$Z = \sum_{k=0}^K A[k] * B[k]$$

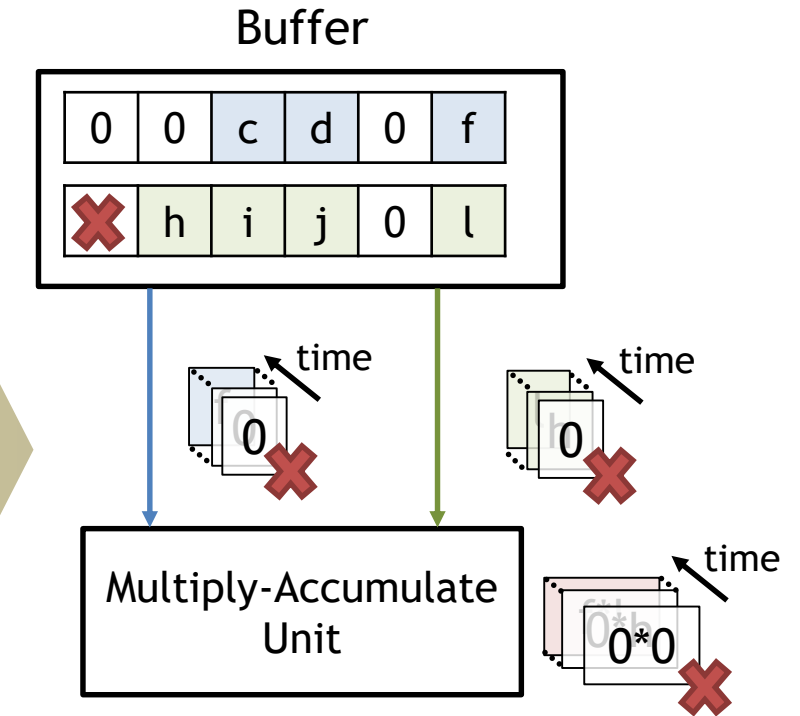


Mapping
Scheduling of data movement & compute in time & space

```
for k in [0:K)  
  Z += A[k] * B[k]
```

Ineffectual computations introduce opportunities to exploit zero-based savings in hardware

Accelerator Architecture



*Z data movements not shown

Hardware Sparse Optimization Features



Format:

Choose tensor representations to save necessary storage spaces and energy associated zero accesses



Gating:

Explicitly eliminate ineffectual storage accesses and computes by letting the hardware unit staying idle for the cycle to save energy



Skipping:

Explicitly eliminate ineffectual storage accesses and computes by skipping the cycle to save energy and time

Various Implementations Lead to Different Performance



Format:

Choose tensor representations to save necessary storage spaces and energy associated zero accesses



Gating:

Explicitly eliminate ineffectual storage accesses and computes by letting the hardware unit staying idle for the cycle to save energy



Skipping:

Explicitly eliminate ineffectual storage accesses and computes by skipping the cycle to save energy and time

What is the chosen format?

Do all tensors share the same format?

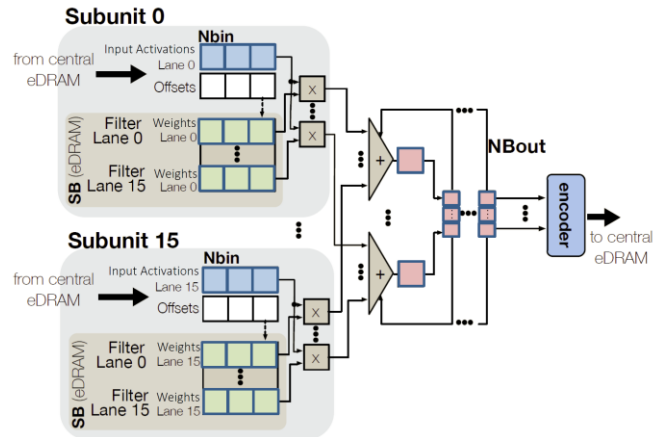
When is a storage access gated?

How much is the compute able to skip ahead?

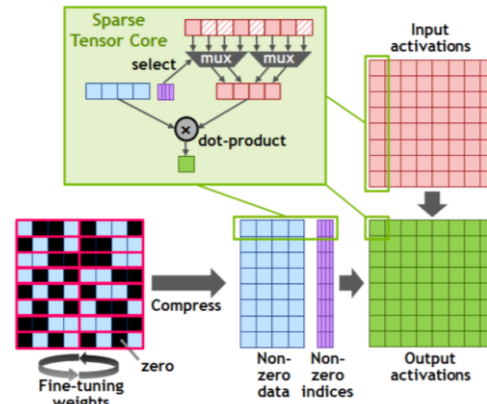
At which storage level is the skipping performed?

What is the criteria for skipping?

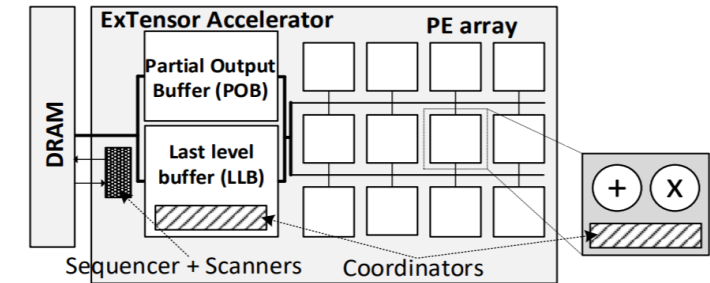
Diverse Sparse Tensor Accelerator Designs



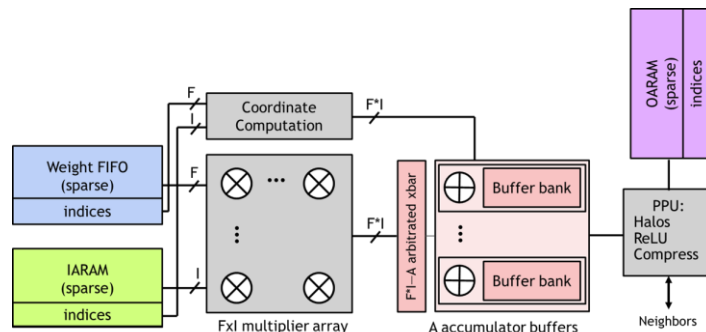
Cnvlutin [ISCA2016]



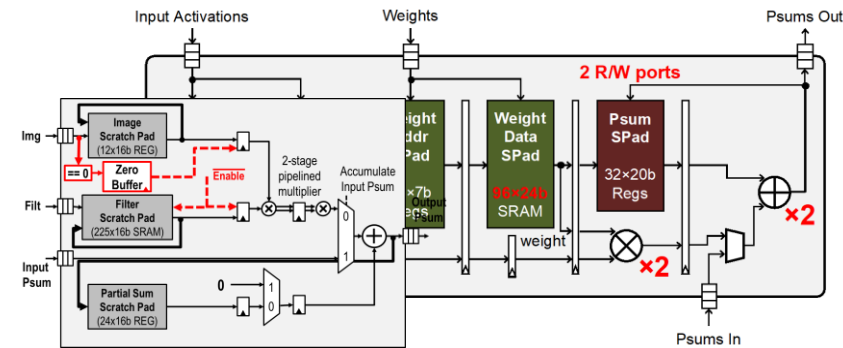
Tensor Core V3 [NVIDIA2020]



ExTensor [MICRO2019]



SCNN [ISCA2017]

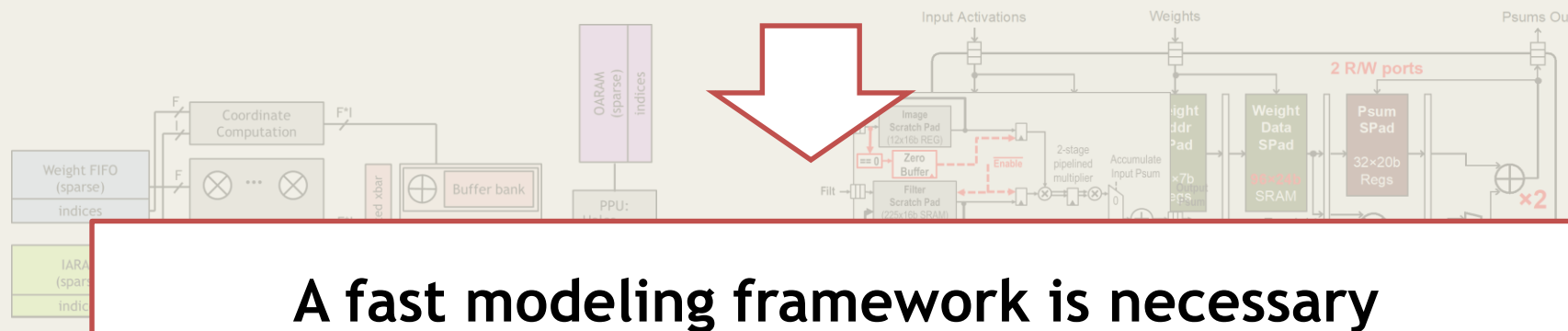
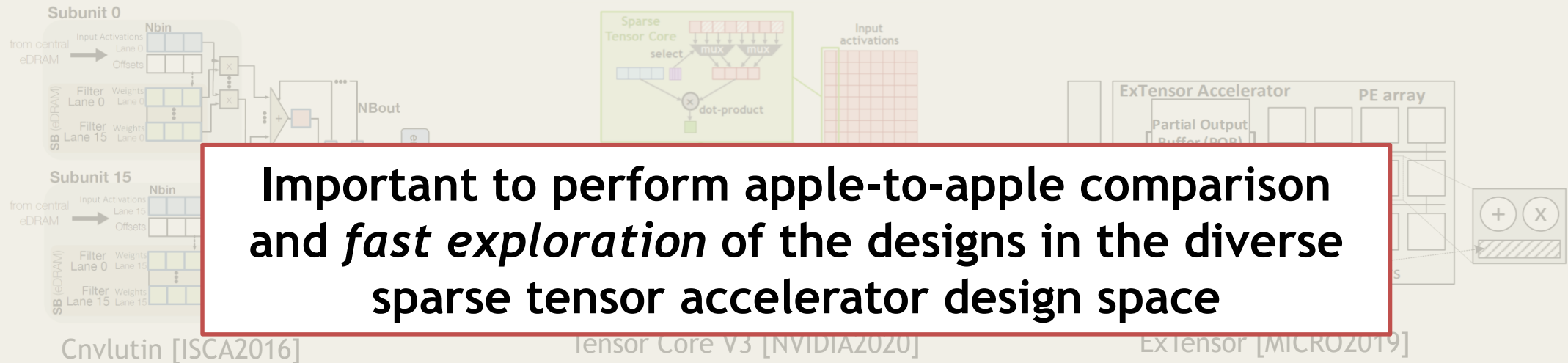


Eyeriss V1 [JSSC 2017]

Eyeriss V2 [JATCAS 2019]

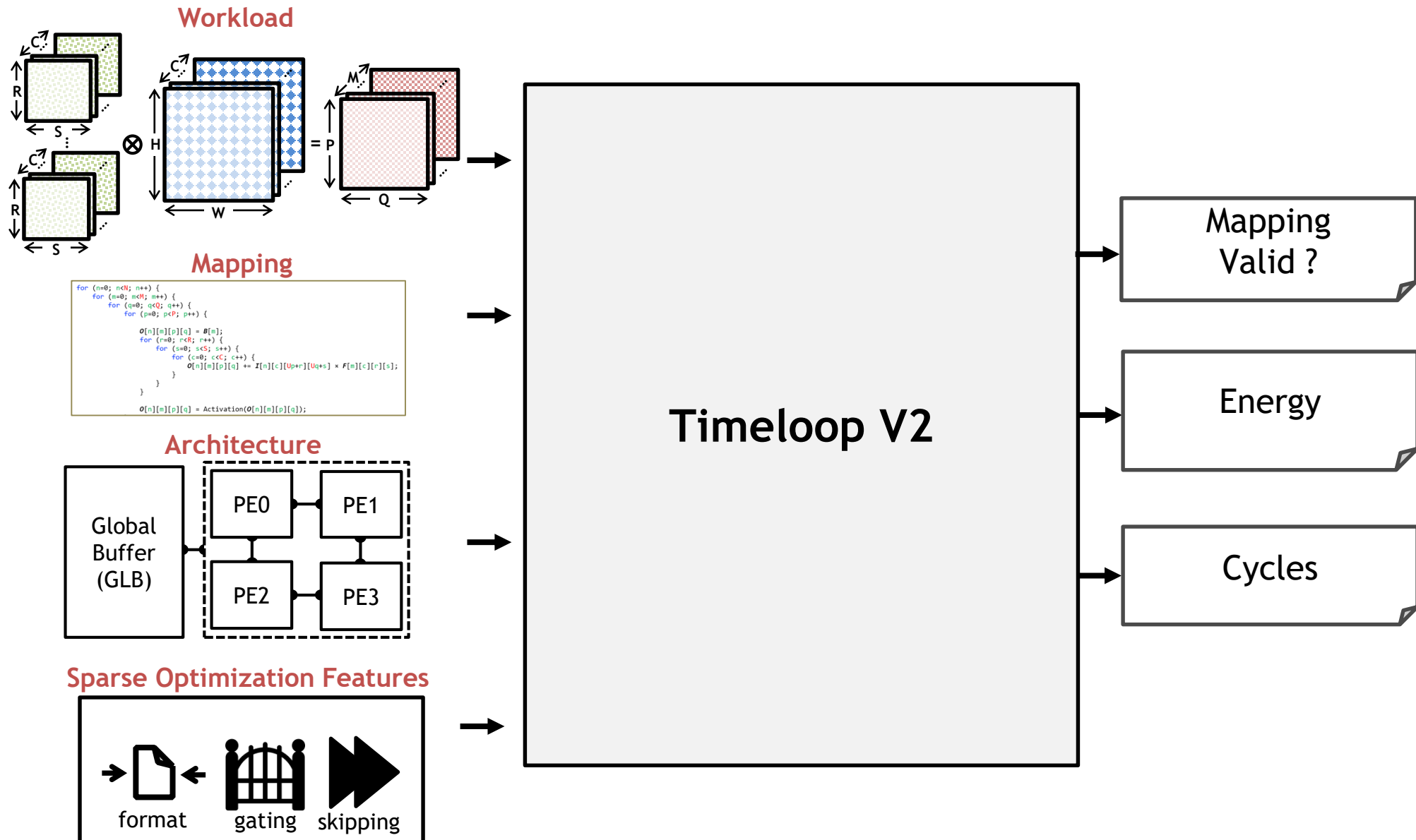
Each accelerator design carefully combines sparse optimization features that work the best with its architecture topology to improve energy efficiency and processing time

Diverse Sparse Tensor Accelerator Designs

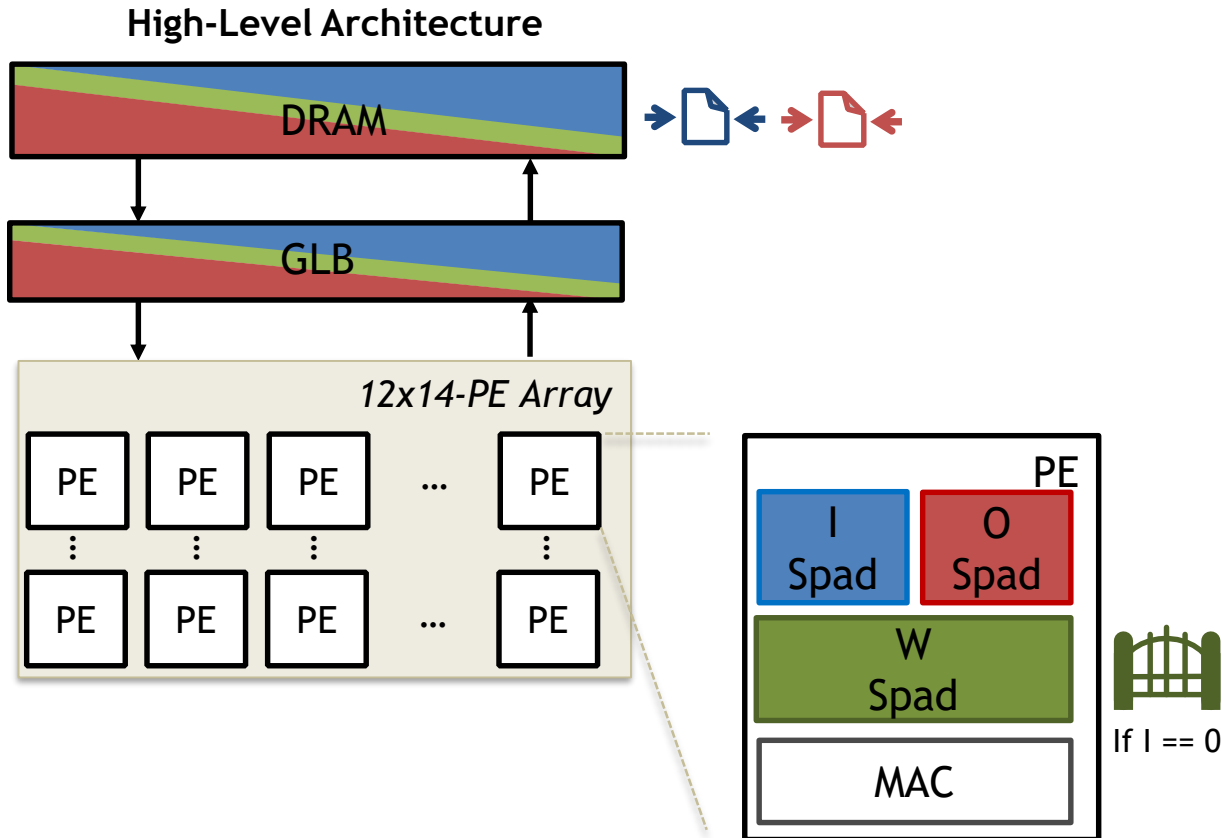


Each accelerator design carefully combines sparse optimization features that work the best with its architecture topology to improve energy efficiency and processing time

Analytical Sparse Tensor Accelerator Modeling



Validation on Eyeriss V1 [ISSCC 2016]



Example Mapping (AlexNet Layer3)
Row Stationary Dataflow

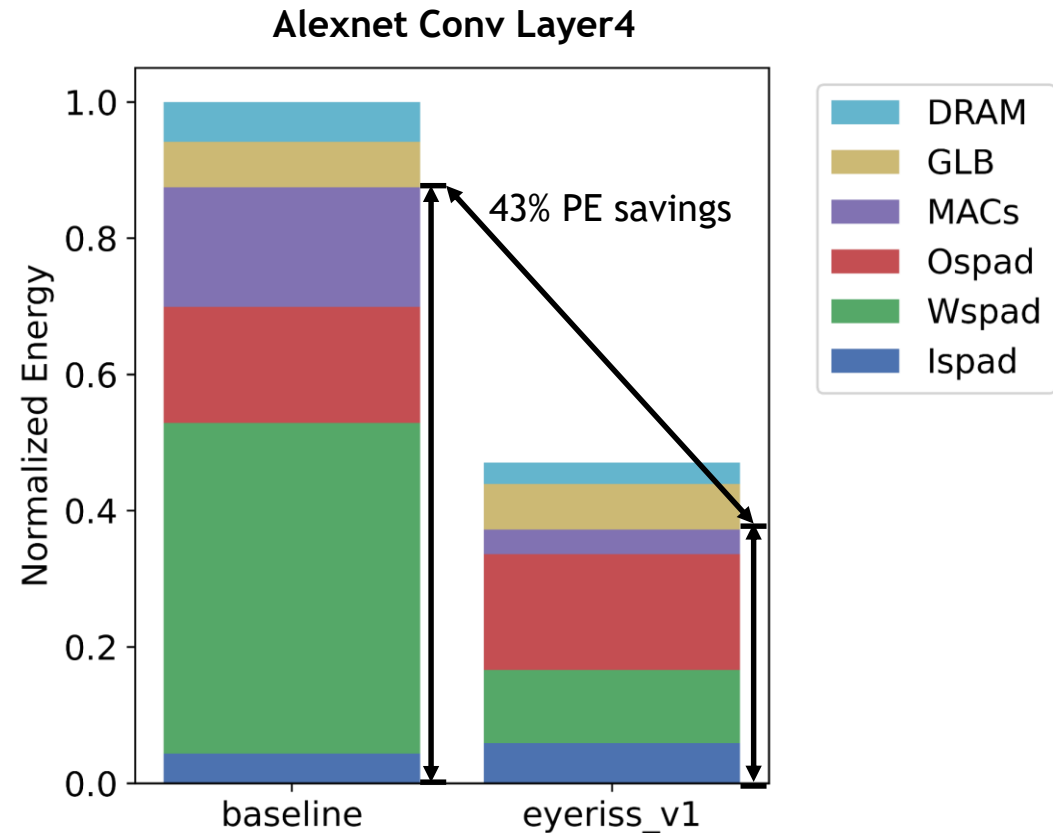
```
DRAM [ Weights:884736 (884736) Inputs:230400 (63361) Outputs:259584 (78654) ]
-----
| for M in [0:6)
|   for C in [0:64)
|
GLB [ Inputs:3600 (3600) Outputs:43264 (43264) ]
-----
|   for N in [0:4)
|   |   for P in [0:13)
|   |   |   for Q in [0:1)
|   |   |   |   for Q in [0:13) (Spatial-X)
|   |   |   |   |   for M in [0:4) (Spatial-Y)
|   |   |   |   |   |   for S in [0:3) (Spatial-Y)
|   |   |   |
ISpad[ Inputs:12 (12) ]
|   |   |   |   for Q in [0:1)
|   |   |   |
WSpad [ Weights:192 (192) ]
|   |   |   |   for R in [0:3)
|   |   |   |   |   for C in [0:4)
|   |   |   |
OSpad [ Outputs:16 (16) ]
|   |   |   |   for M in [0:16)
```

Validation on Eyeriss V1 [ISSCC 2016]

- DRAM compression ratio

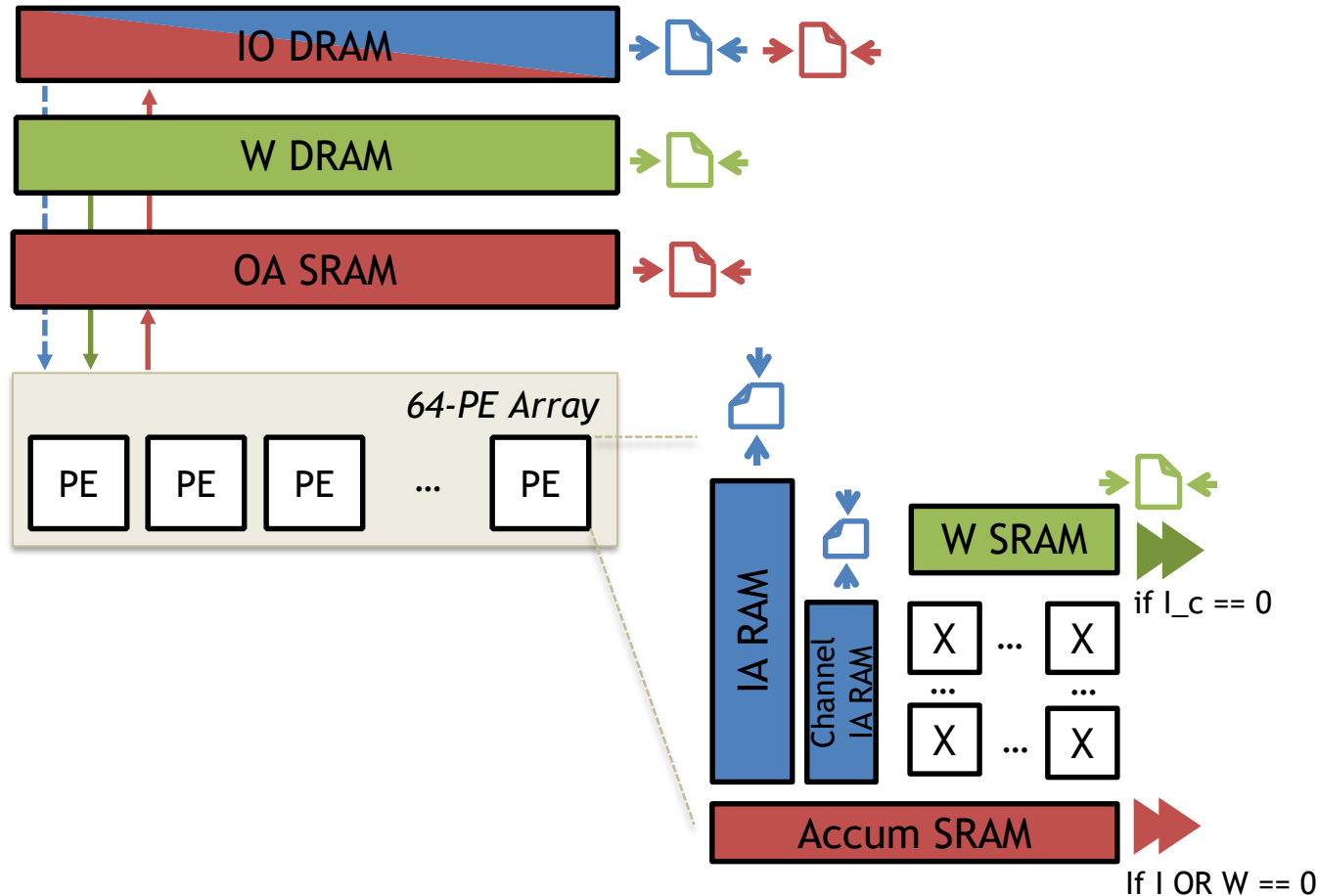
layer	Eyeriss	our work
1	1.2	1.24
2	1.4	1.37
3	1.7	1.68
4	1.8	1.86
5	1.9	1.93

- Normalized energy consumption with sparse optimization applied
 - 45% vs. 43% in our estimation, 96% accurate



Validation on SCNN Architecture [ISCA2017]

High-Level Architecture



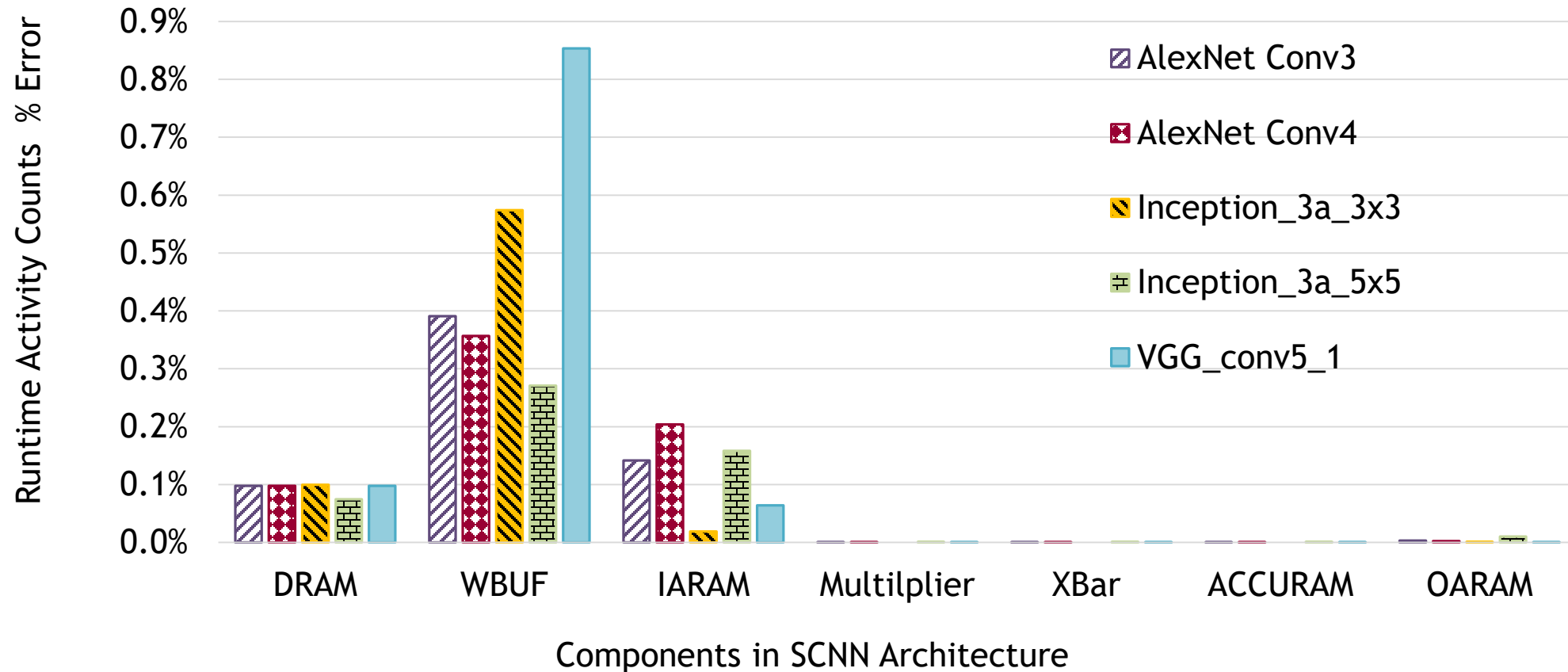
Example Mapping (AlexNet Layer3)
Input Stationary Cartesian Product Dataflow

```

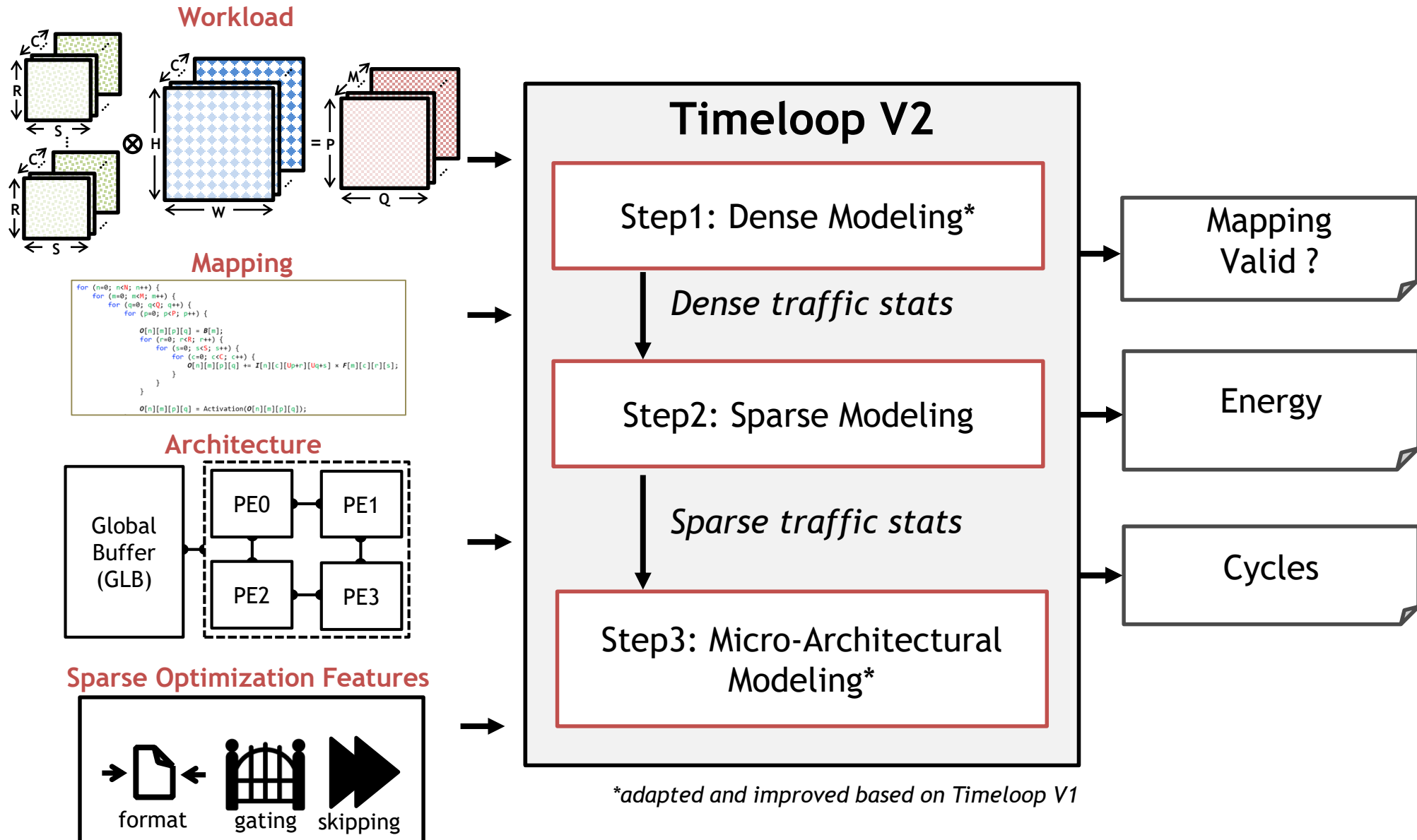
IO DRAM [ ]
-----
| for W in [0:1)
O ARAM [ Outputs:75264 (34742) ]
-----
|   for W in [0:1)
W DRAM [ Weights:884736 (325761) ]
-----
|     for M in [0:6)
|     for W in [0:6) (Spatial-X)
|     for H in [0:6) (Spatial-X)
IA RAM [ Inputs:1024 (639) ]
-----
|   for W in [0:1)
Accumu SRAM [ Outputs:1024 (1024) ]
-----
|     for C in [0:256)
Channel IARAM [ Inputs:4 (4) ]
-----
|       for W in [0:1)
W SRAM [ Weights:576 (213) ]
-----
|         for M in [0:16)
|         for S in [0:3)
|         for R in [0:3)
|         for M in [0:4) (Spatial-Y)
|         for W in [0:2) (Spatial-X)
|         for H in [0:2) (Spatial-X)
    
```

Validation on SCNN Architecture [ISCA2017]

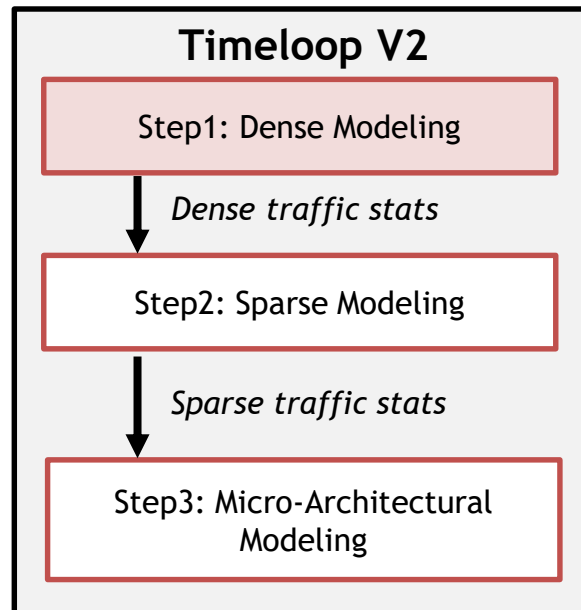
Less than 1% error comparing to results generated by a custom SCNN simulator



Proposed Analytical Sparse Tensor Accelerator Modeling

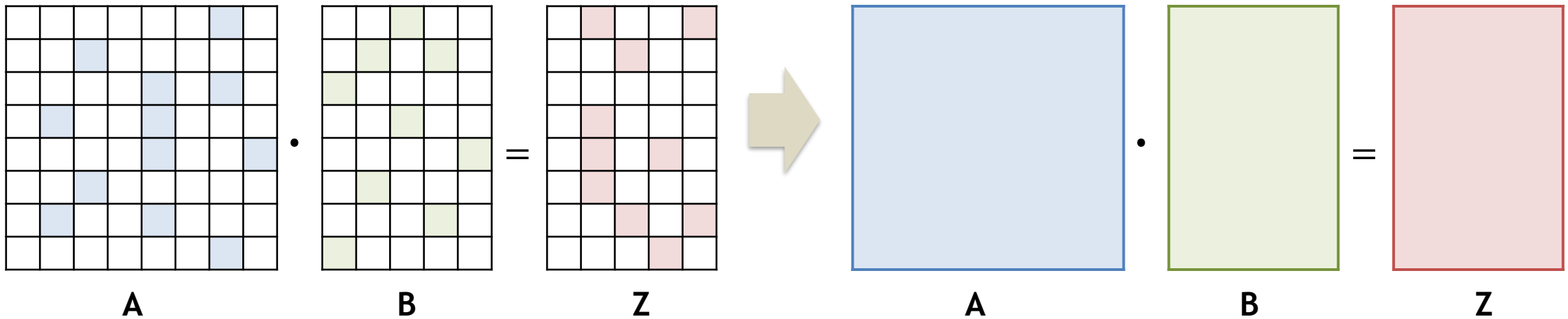


Analytical Modeling for Dense Accelerators



Abstracts Problem Instance Details Away

Fast analytical modeling does not examine the exact data in workloads

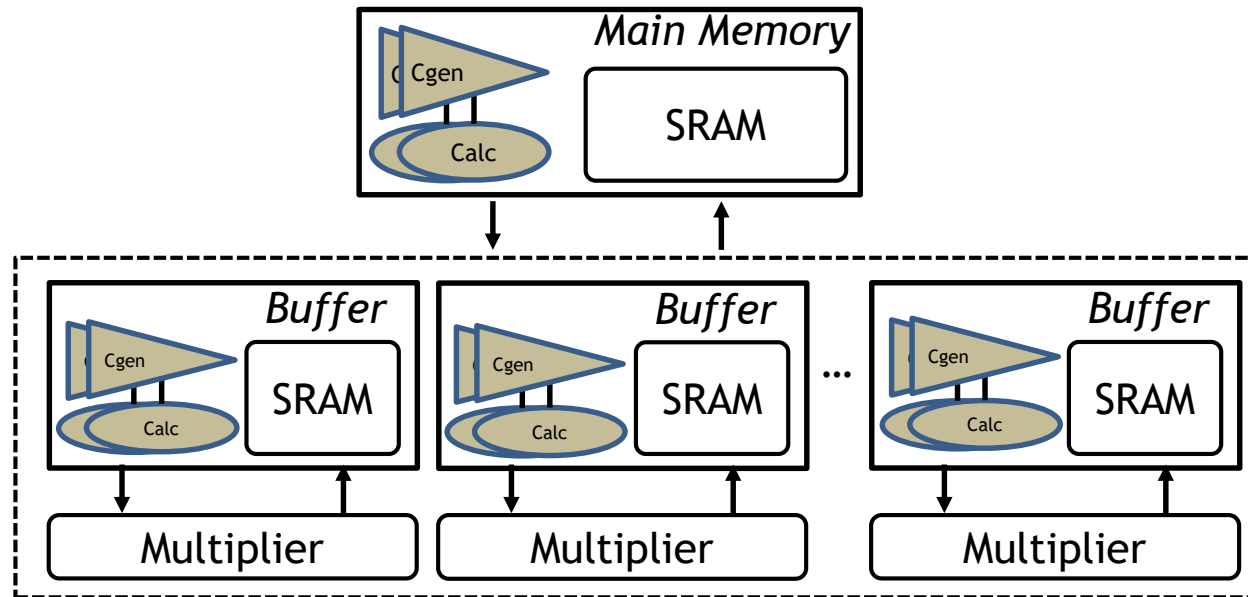


Exact Problem Instance

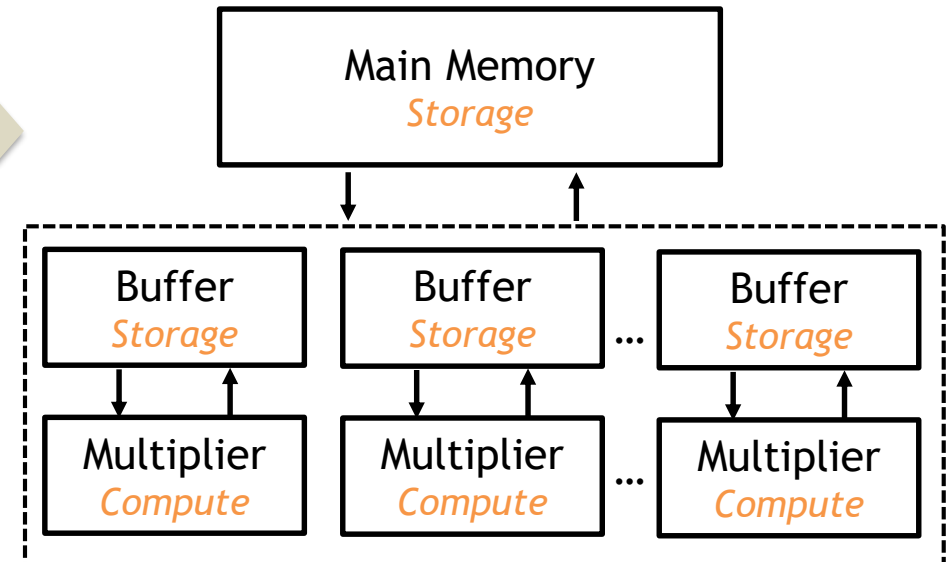
Problem Instance Shapes

Abstracts Architecture Details Away

Fast analytical modeling does not examine detailed architecture implementation



Detailed Architecture

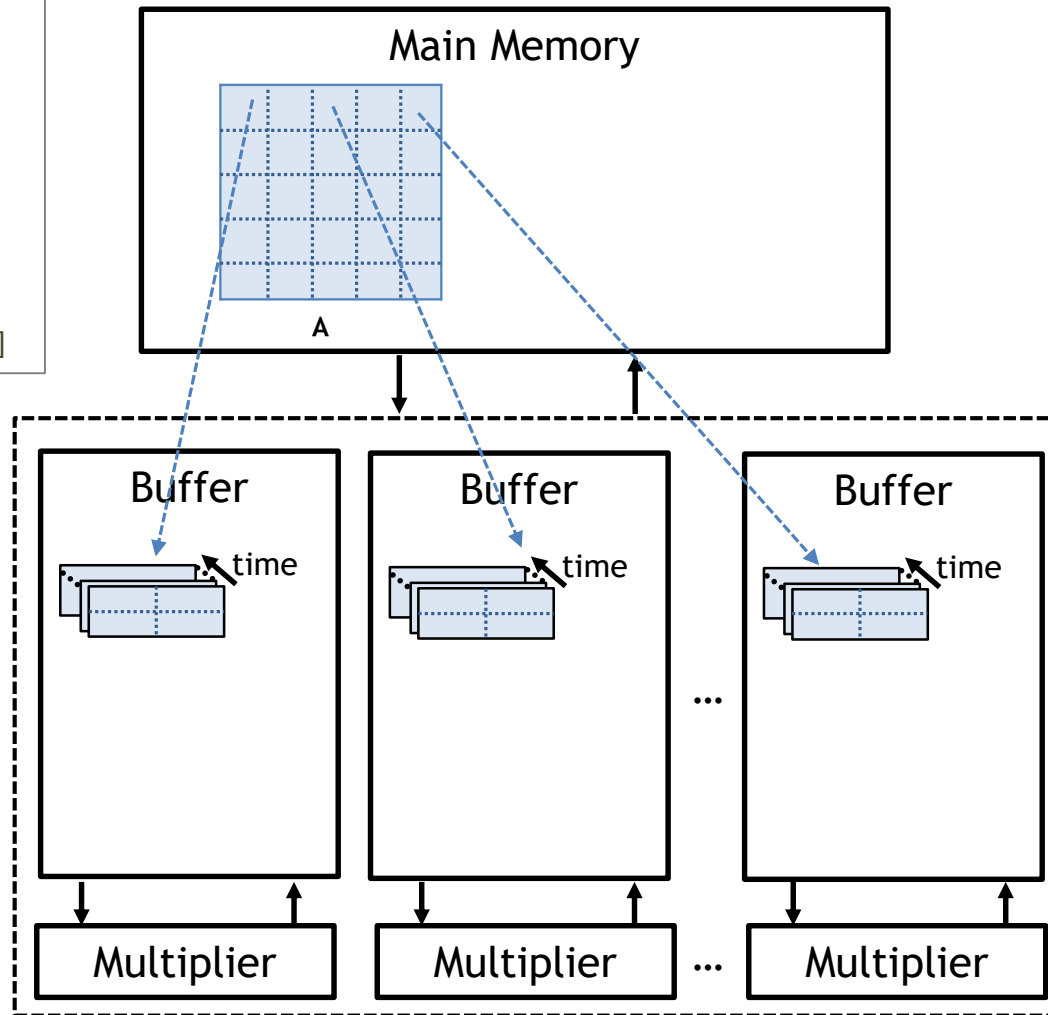


Abstract Architecture Topology

Dense Data Movement and Compute Analysis

Example Mapping

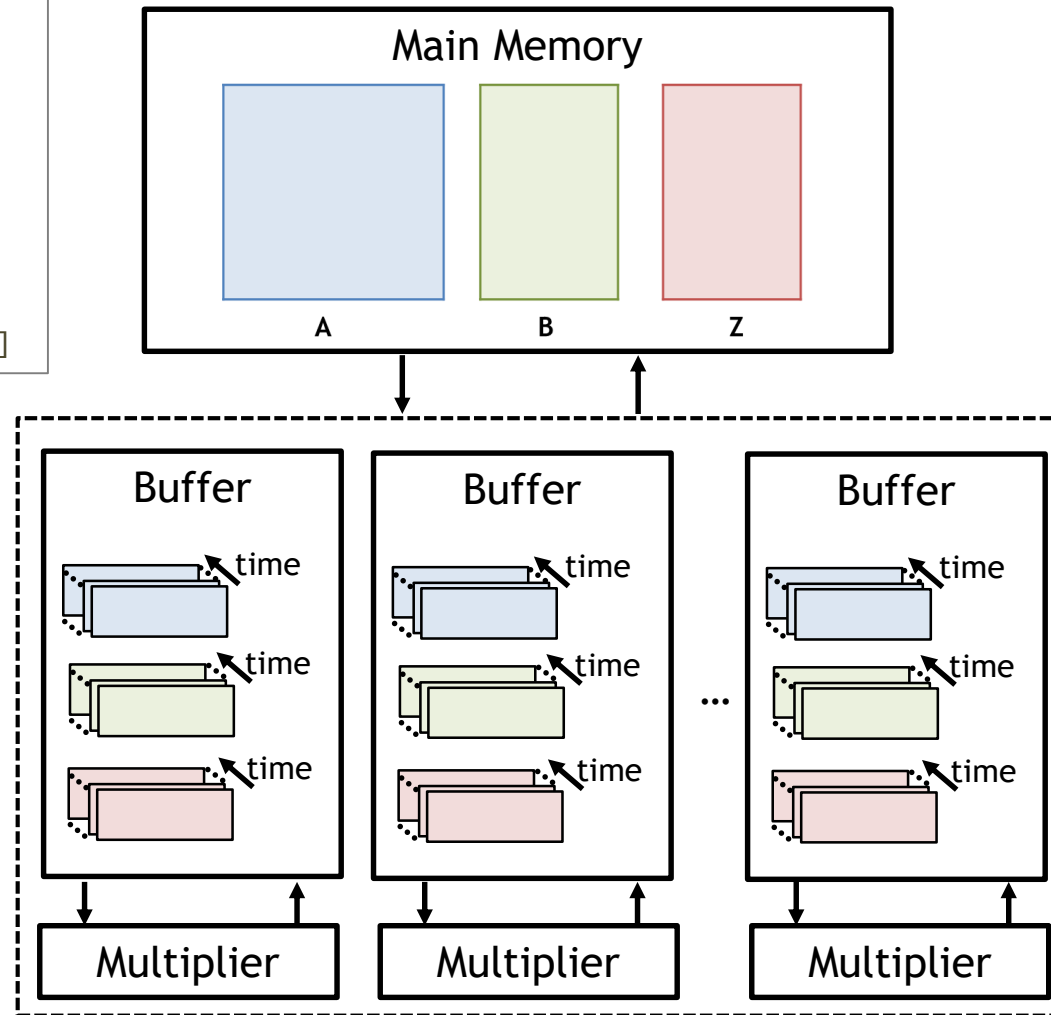
```
----- Main Memory -----  
for m in [0:M2)  
  for n in [0:N2)  
    for k in [0:K2)  
      par-for m in [0:M1)  
      par-for n in [0:N1)  
      par-for k in [0:K1)  
----- Buffer-----  
  for m in [0:M0)  
    for n in [0:N0)  
      for k in [0:K0)  
        Z[m,n] += A[m,k]*B[k,n]
```



Dense Data Movement and Compute Analysis

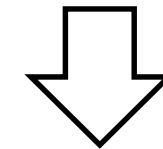
Example Mapping

```
----- Main Memory -----  
for m in [0:M2)  
  for n in [0:N2)  
    for k in [0:K2)  
      par-for m in [0:M1)  
      par-for n in [0:N1)  
      par-for k in [0:K1)  
----- Buffer-----  
  for m in [0:M0)  
    for n in [0:N0)  
      for k in [0:K0)  
        Z[m,n] += A[m,k]*B[k,n]
```



Answer dataflow related questions

- Which tensor is temporally reused at each storage level?
- How much data is transferred between storages?
- How many compute happened?
- ...

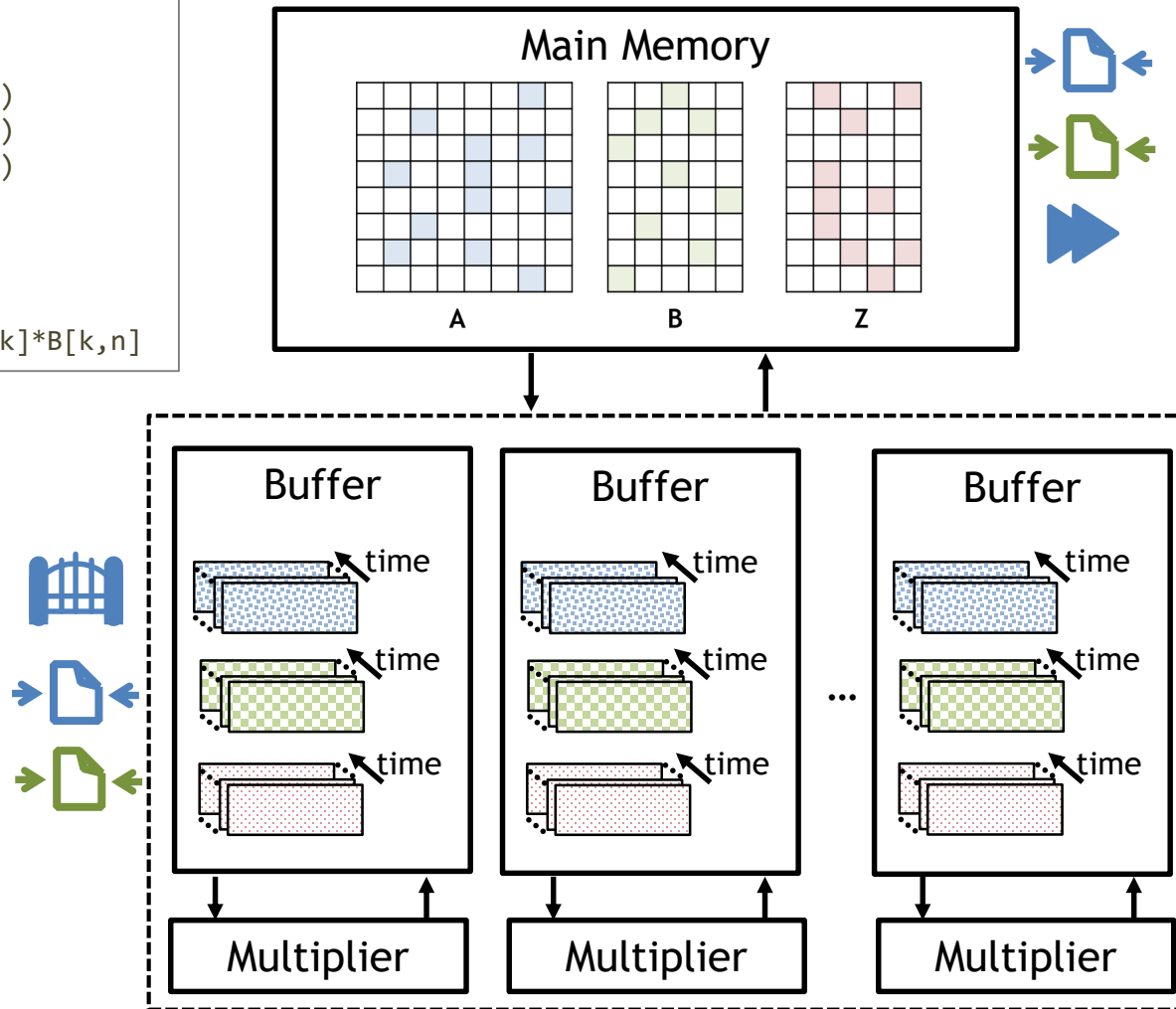


Mapping Valid?
Energy Efficiency
Cycles

Sparse Accelerator Modeling is Data Dependent

Example Mapping

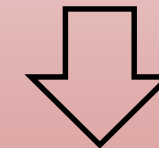
```
----- Main Memory -----  
for m in [0:M2)  
  for n in [0:N2)  
    for k in [0:K2)  
      par-for m in [0:M1)  
      par-for n in [0:N1)  
      par-for k in [0:K1)  
    ----- Buffer-----  
    for m in [0:M0)  
      for n in [0:N0)  
        for k in [0:K0)  
           $Z[m,n] += A[m,k]*B[k,n]$ 
```



What is impact of sparse optimization features?

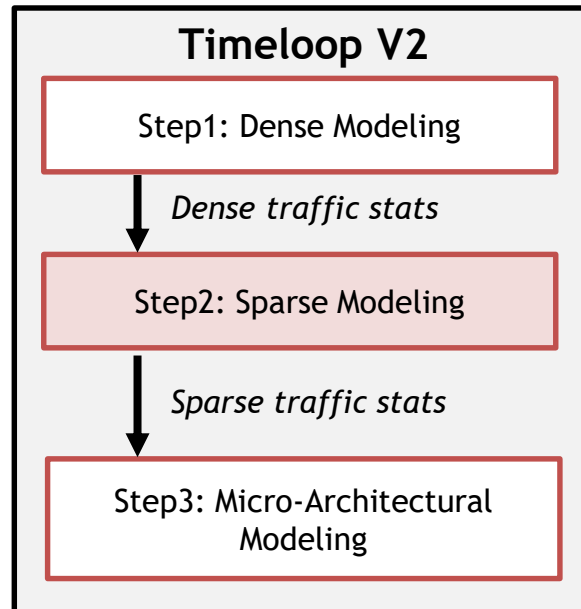
Answer dataflow related questions

- Which tensor is temporally reused at each storage level?
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- ...

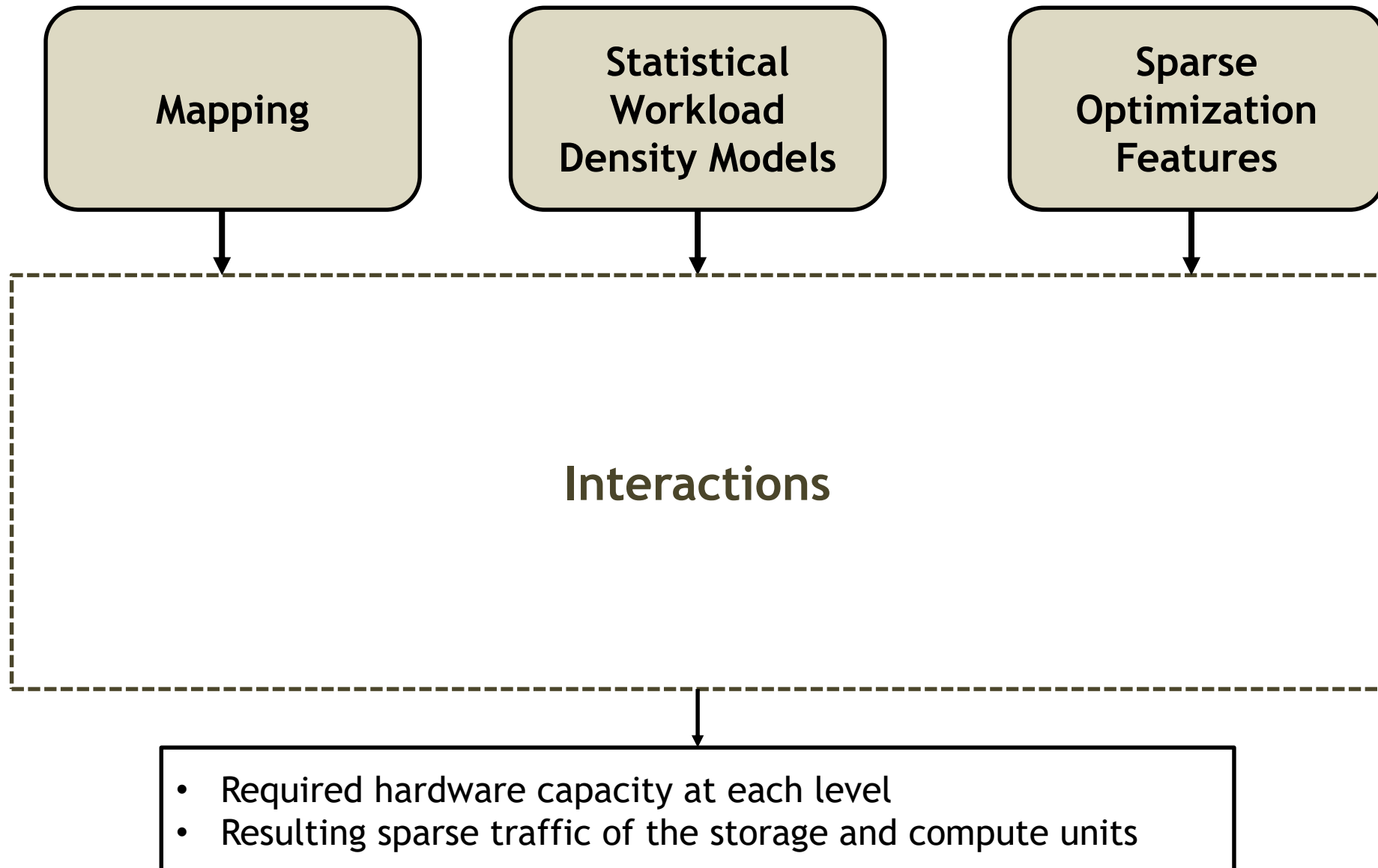


Mapping Valid?
Energy Efficiency
Cycles

Proposed Sparse Tensor Accelerator Modeling Methodology



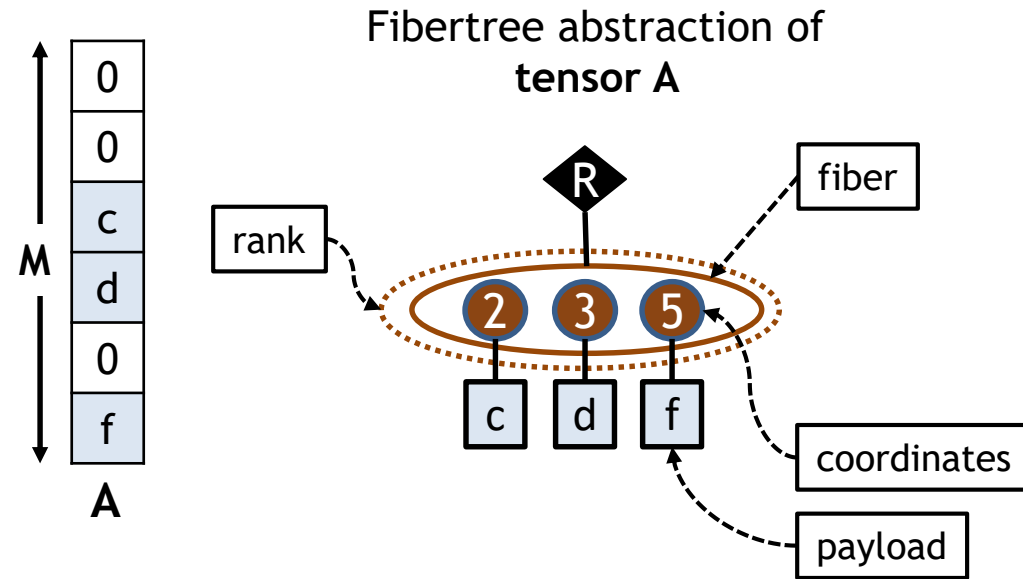
Specifications and Their Interactions



Proposed Sparse Tensor Accelerator Modeling Methodology

Interactions Between Mapping and Workload Density Models

Analysis Based on Fibertree-based Tensor Abstraction

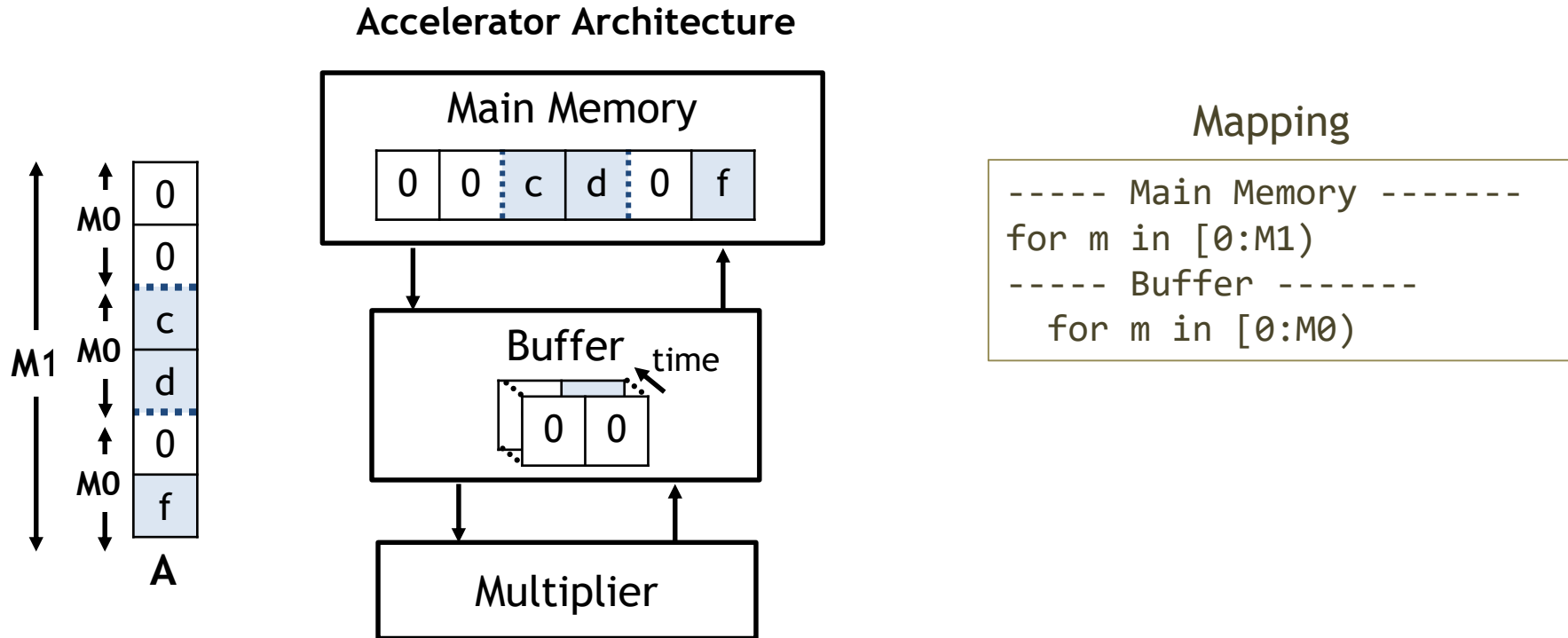


The format-agnostic nature of fibertree allows clean separation of the sparse nature of tensor and its format

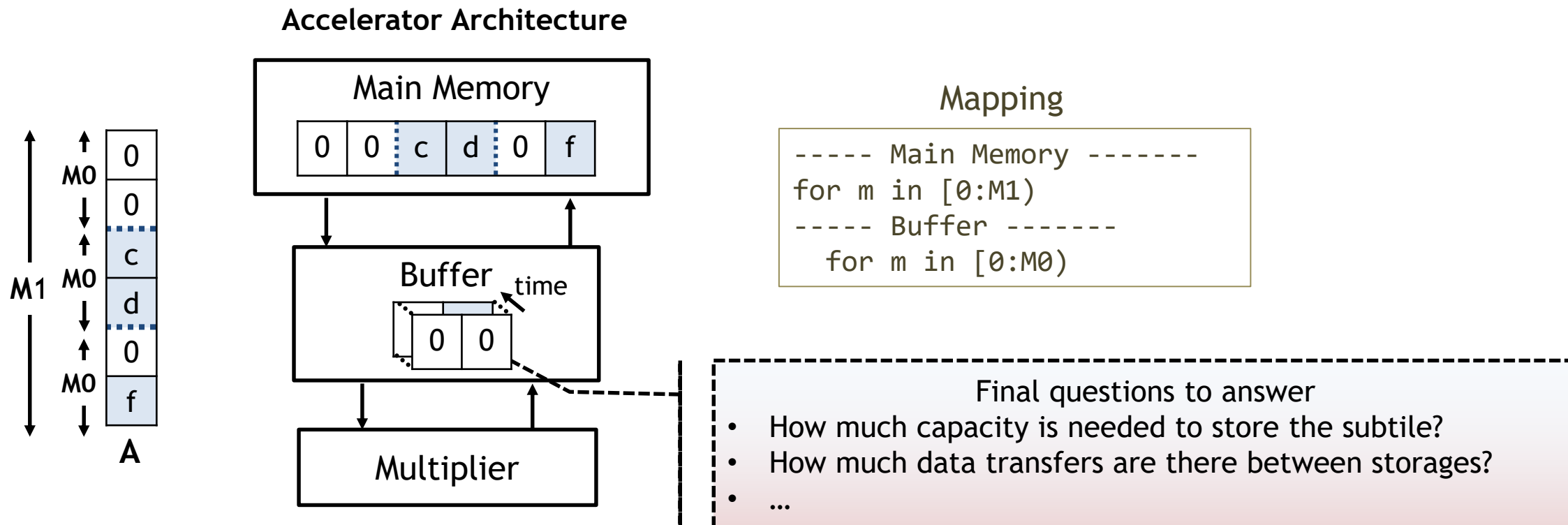
Decides the theoretical savings sparse optimization features can bring

One of the implementation decisions to realize sparse optimization features

Mapping Introduces Tiled Tensors

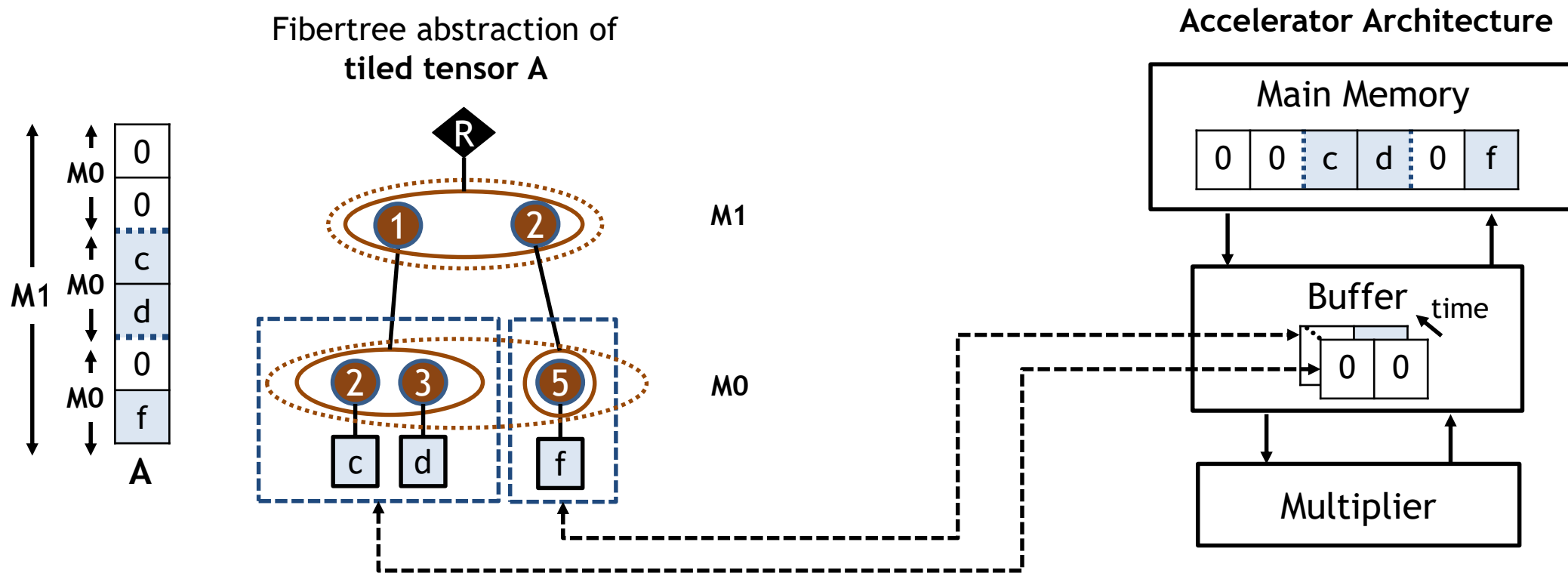


Mapping Introduces Tiled Tensors



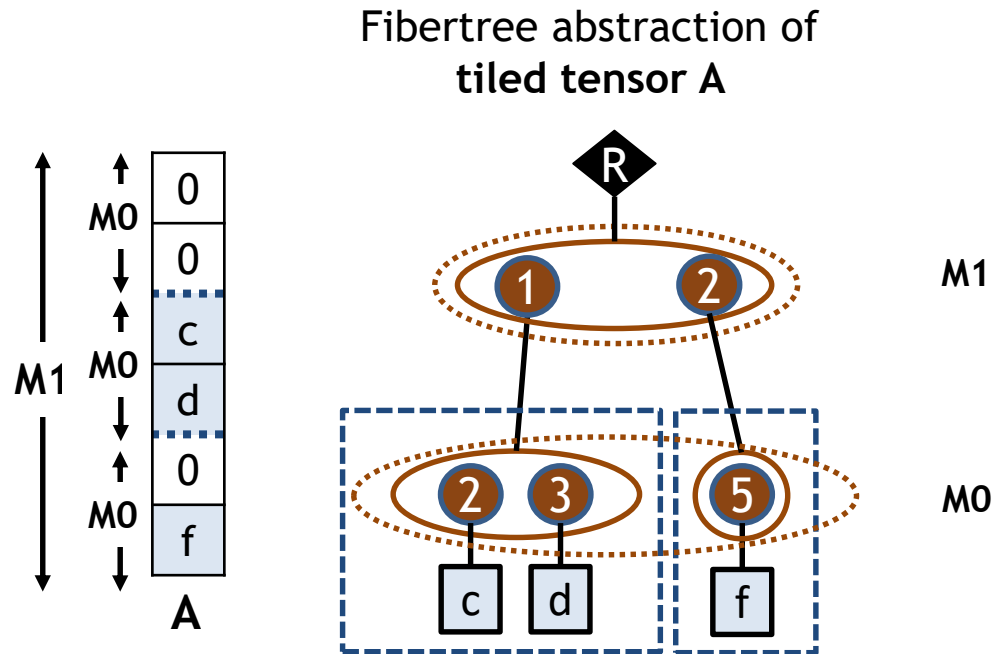
All dependent on the sparse nature of the (sub)tensor, i.e., how many nonzeros values in (sub)tensor

Fibertree Defines the Sparse Nature of Tensors



Characterizing the sparse nature of a (sub)tensor
==
Characterizing a fiber

Fibertree Defines the Sparse Nature of Tensors



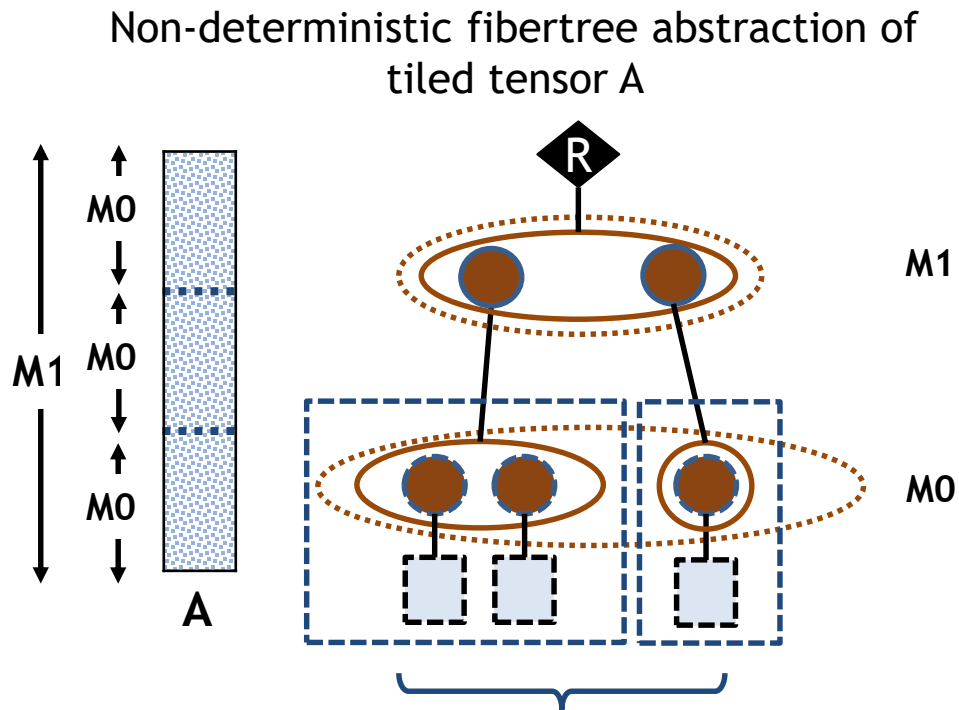
To characterize all the fibers in the tensor, we need to consider

- # of ranks
- # of fibers in each rank
- # of elements in each fiber, i.e., fiber occupancy

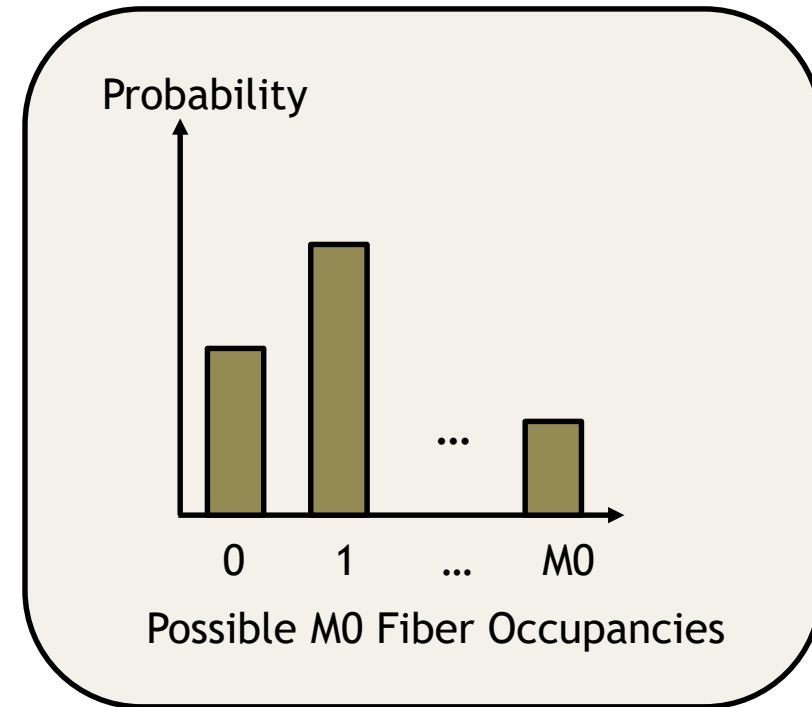
Deterministic when exact data can be examined

Statistical Density Models Necessary for Analytical Modeling

To ensure fast modeling speed, analytical modeling cannot examine the exact data in fibers



Without exact data, the # of fibers and # of elements in each fiber cannot be determined

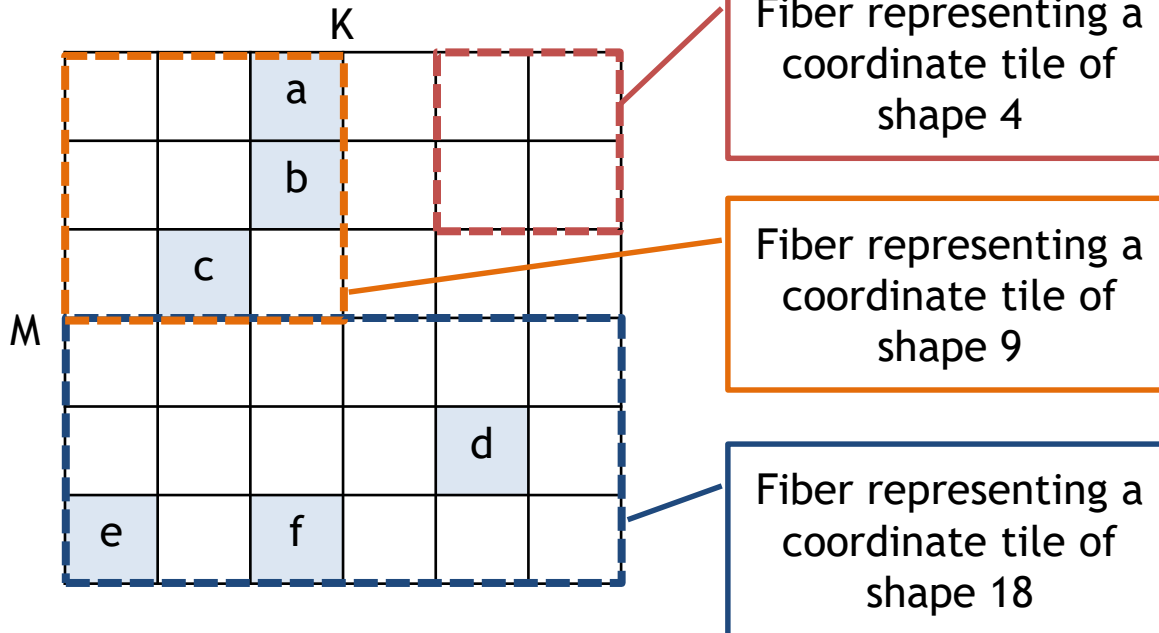


Probability distributions depend on the choice of statistical workload density model

Density Model 1: Hypergeometric Distribution

Describes the randomly distributed zeros in a tensor

Example 6x6 tensor with
randomly distributed density of $1/6$

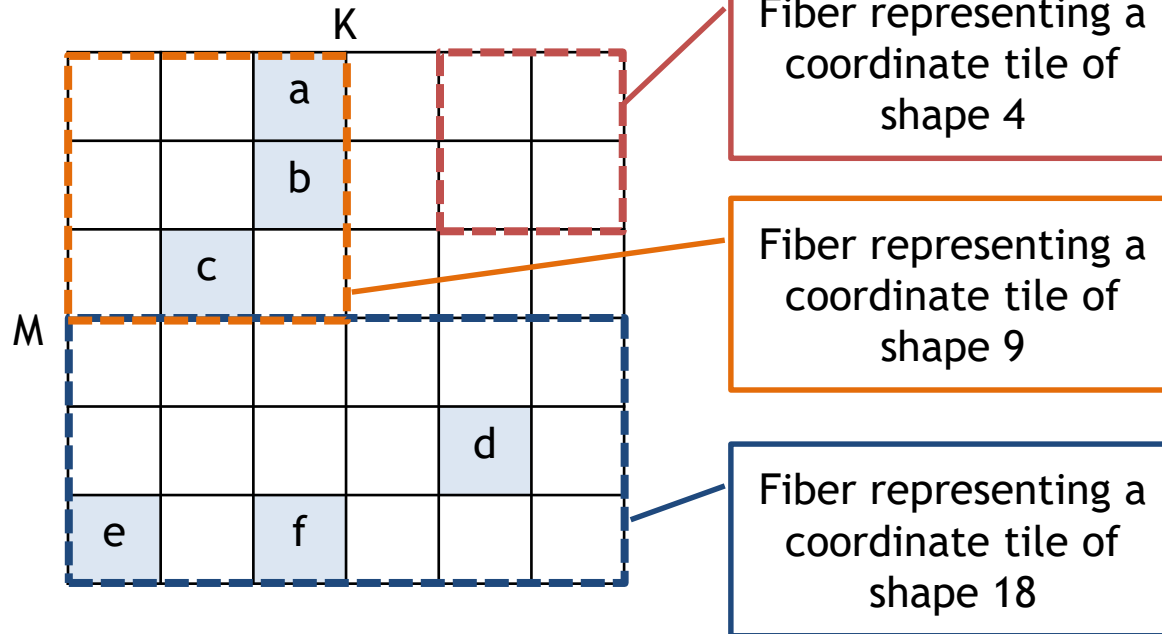


Main Characteristics

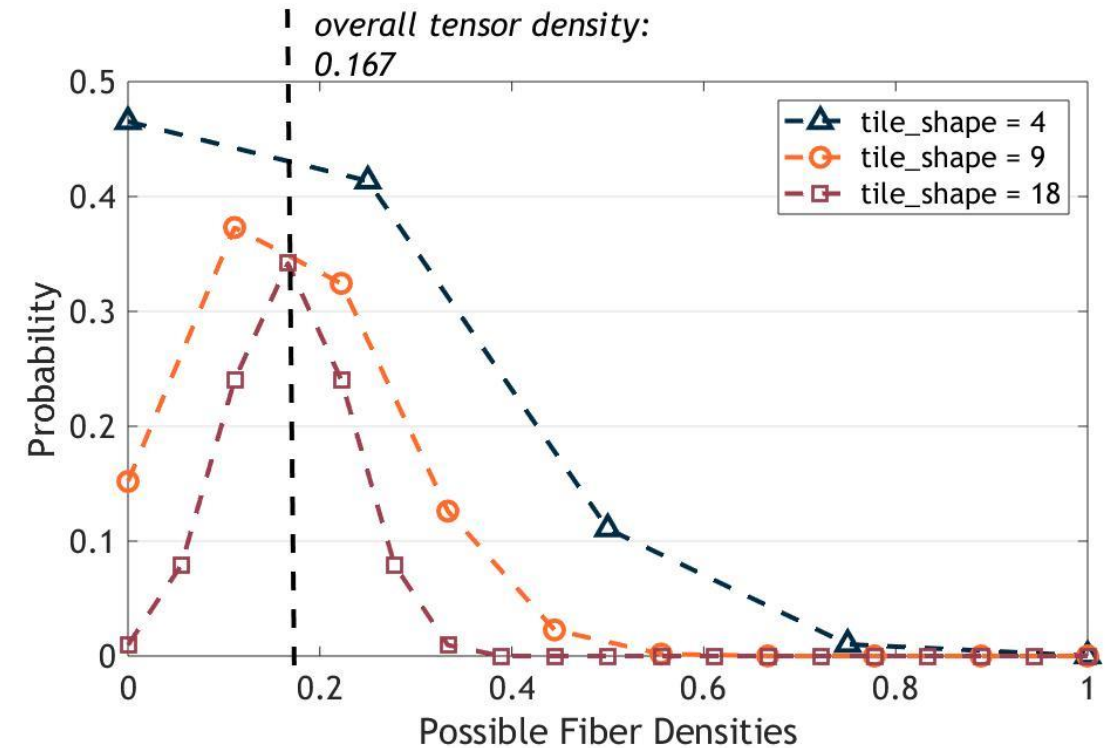
The smaller the tile is, the more likely for the fiber to be empty/full (low density/high density)

Density Model 1: Hypergeometric Distribution

Example 6x6 tensor with randomly distributed density of $1/6$



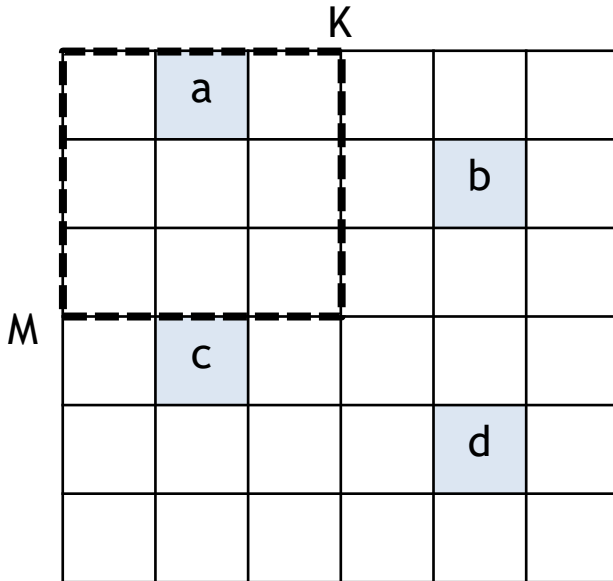
Fiber Densities Characterized By Hypergeometric Model



Density Model 2: Fixed-Structured Distribution

Describes a structured distribution of zeros in a tensor, where all tiles in the tensor have a shared fixed density

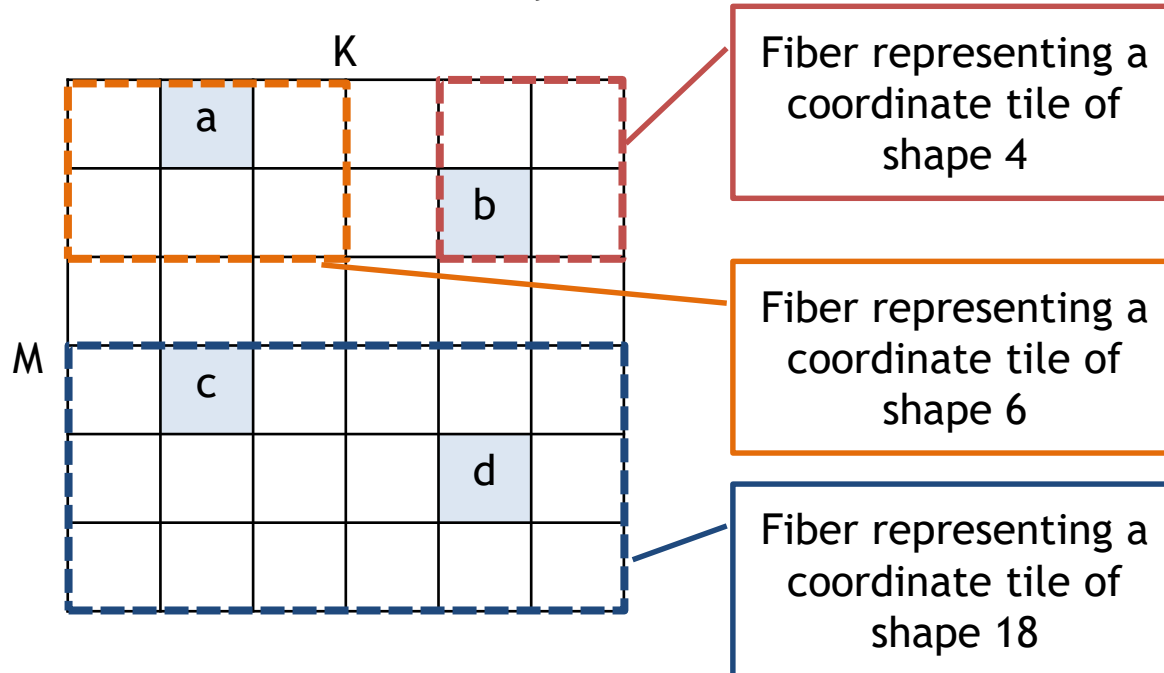
Example 6x6 tensor with
a fixed structured density of $1/9$



Density Model 2: Fixed-Structured Distribution

Describes a structured distribution of zeros in a tensor, where all tiles in the tensor have a shared fixed density

Example 6x6 tensor with a fixed structured density of 1/9



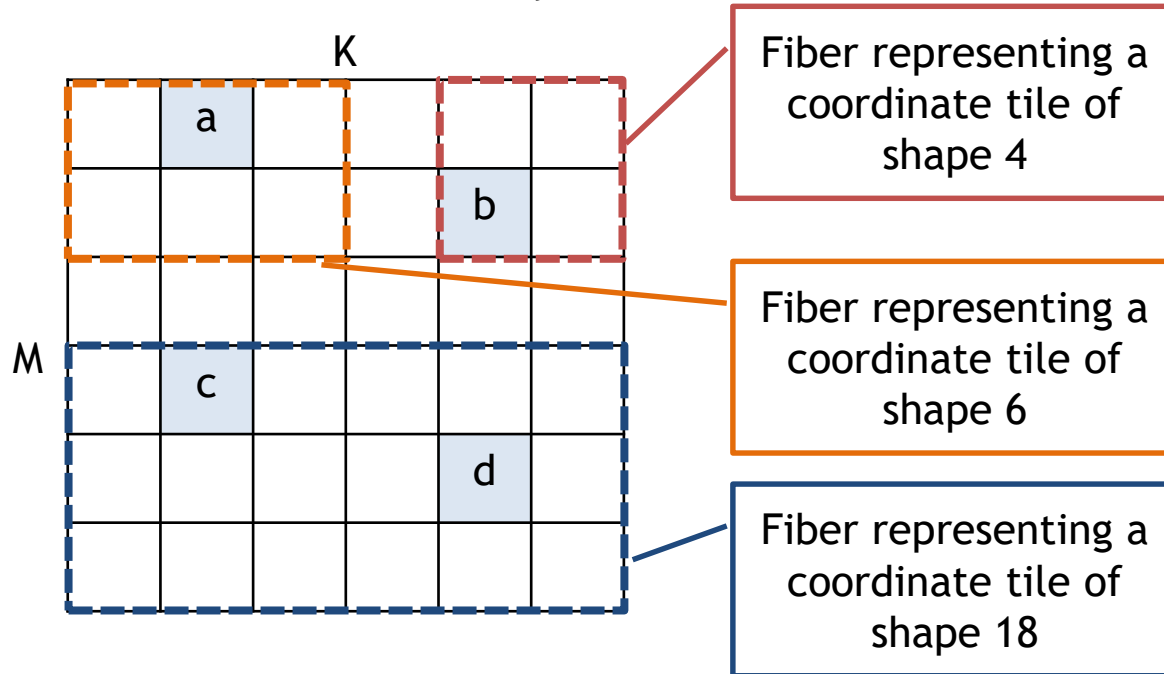
Main Characteristics

Fibers might have non-deterministic occupancy if tile *shape* \times *fixed density* is non-integer

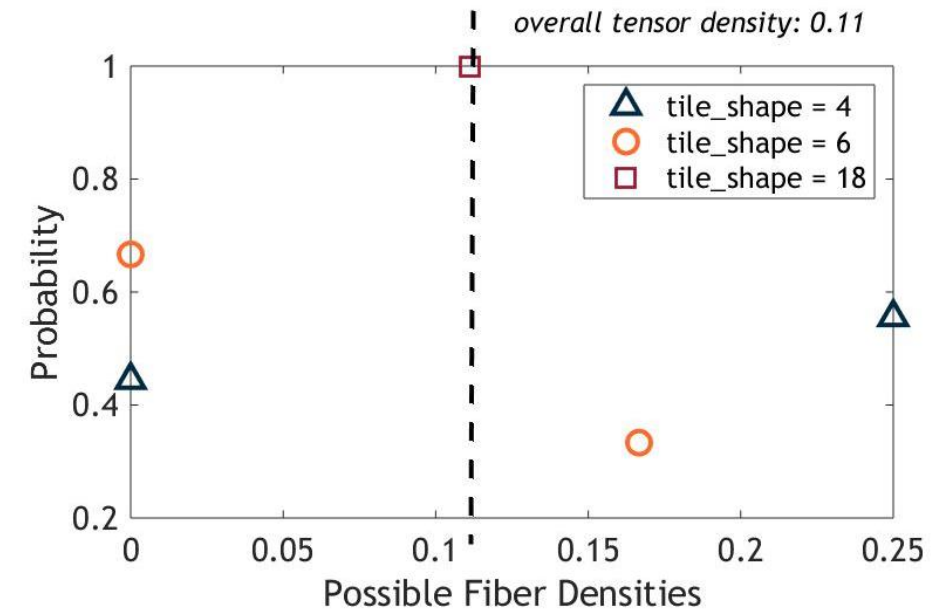
Density Model 2: Fixed-Structured Distribution

Non-integer occupancy represented as weighted sum of integer possible occupancies

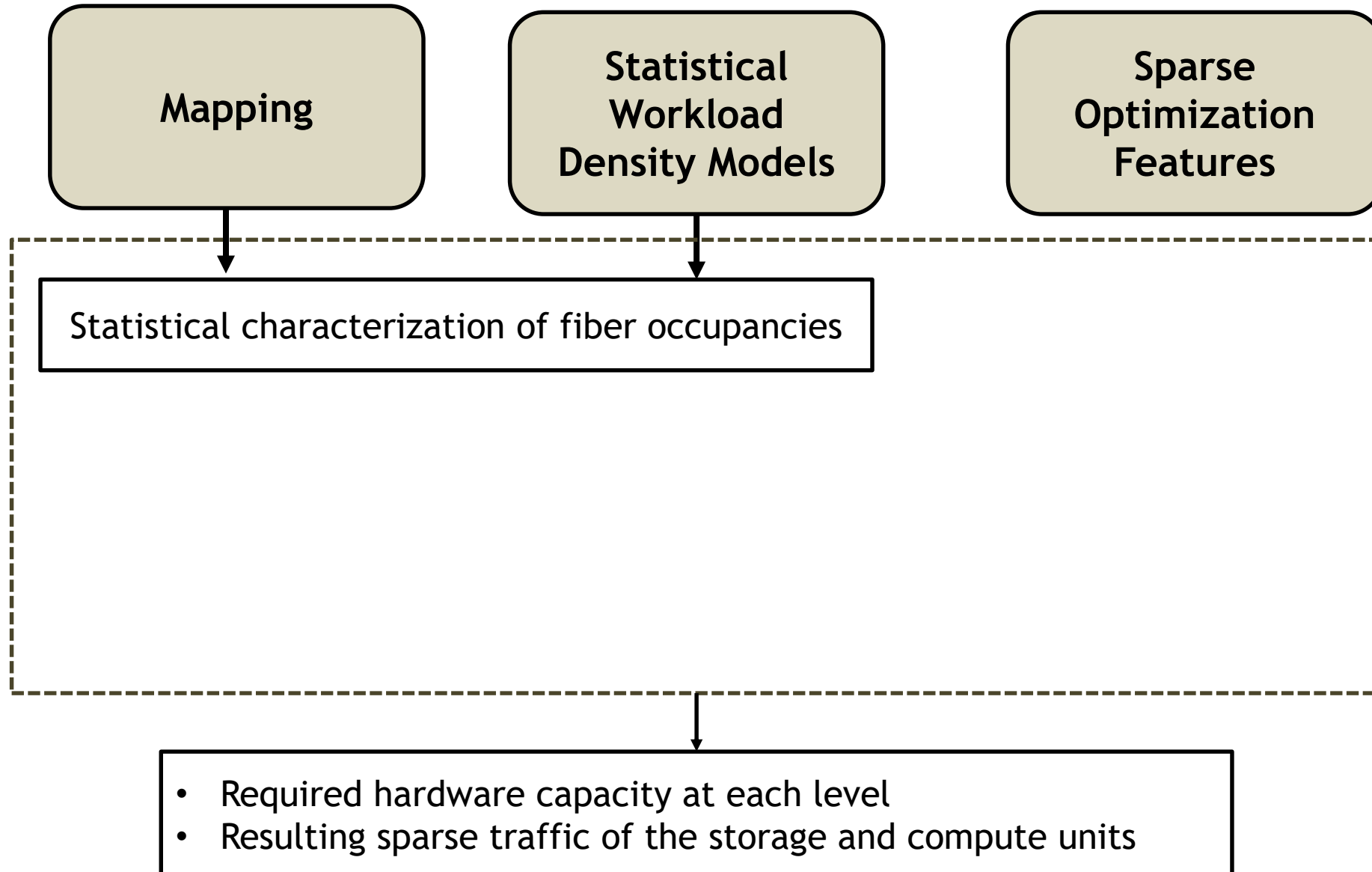
Example 6x6 tensor with
a fixed structured density of 1/9



Fiber Densities Characterized By
Fixed-Structured Density Model



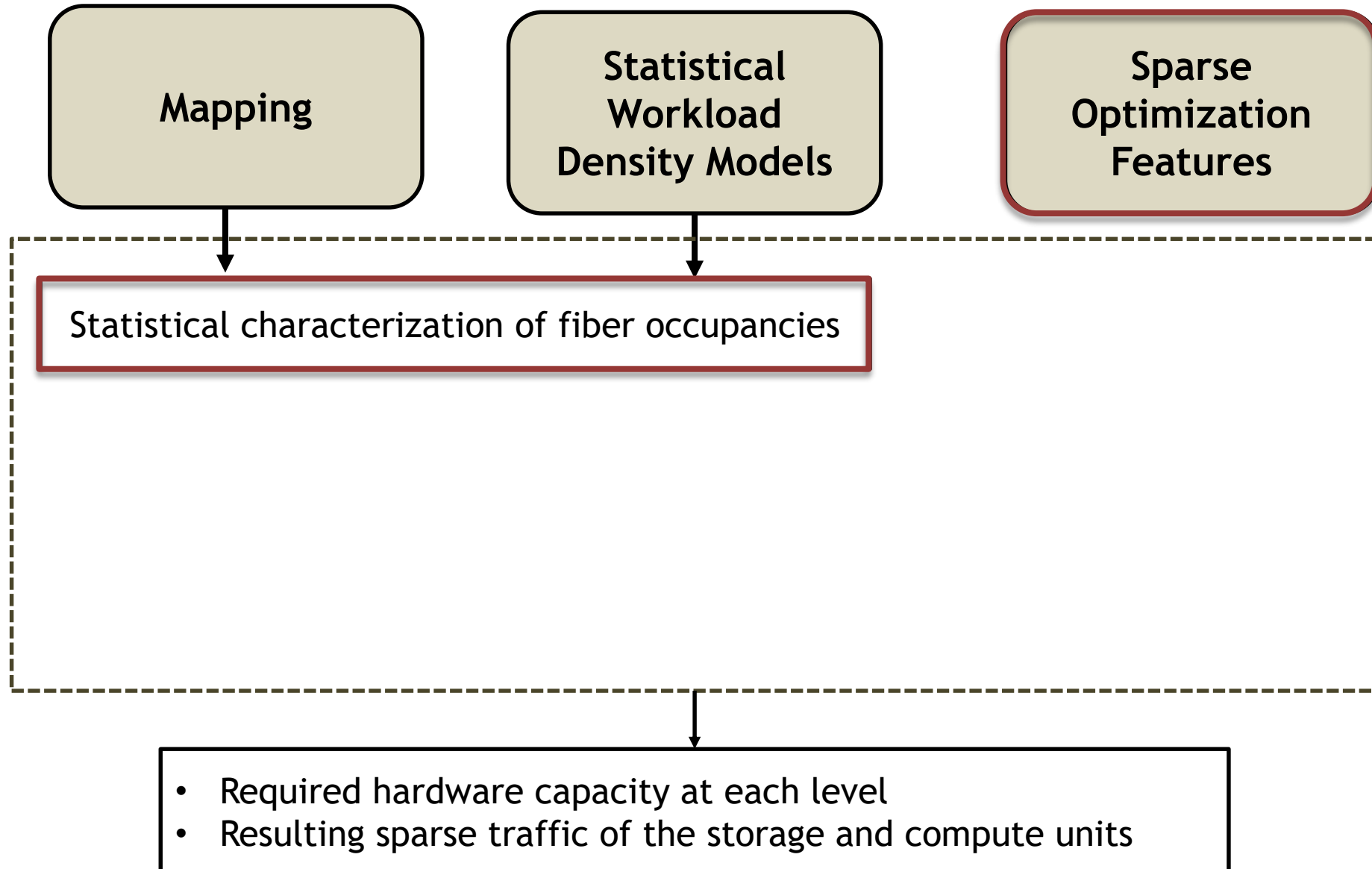
Specifications and Their Interactions



Proposed Sparse Tensor Accelerator Modeling Methodology

Sparse Optimization Feature Impact Modeling

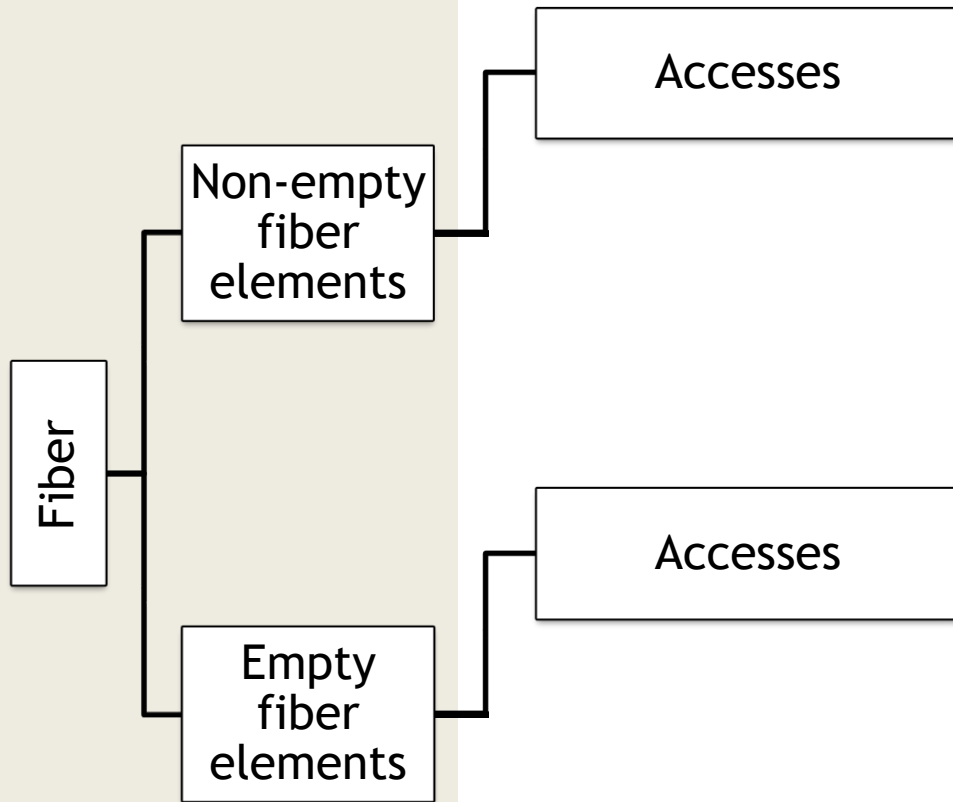
Specifications and Their Interactions



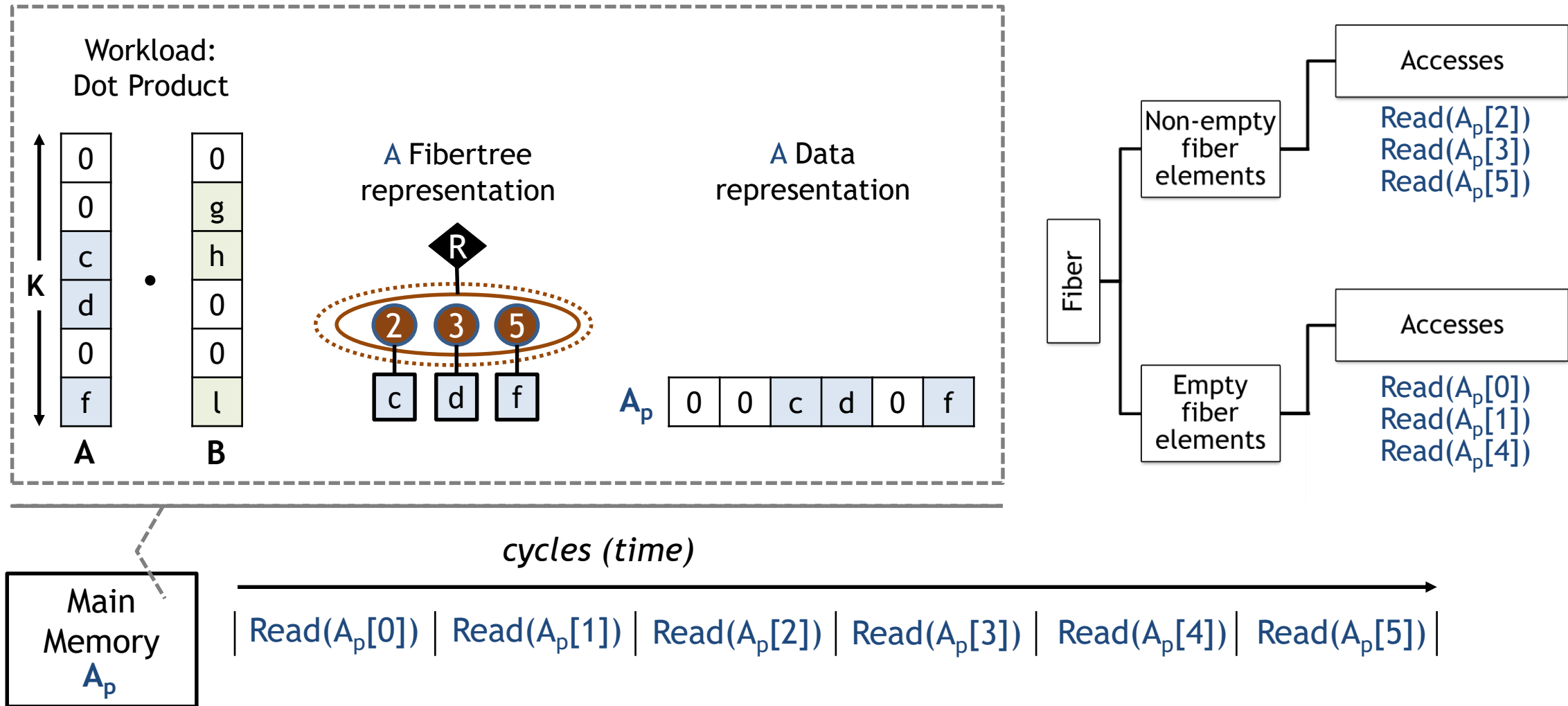
Interactions

Baseline Storage Access Types Related to a Fiber

Deterministic based
on the statistical
occupancy of fiber



Baseline A Tensor Accesses in A Dot Product Workload

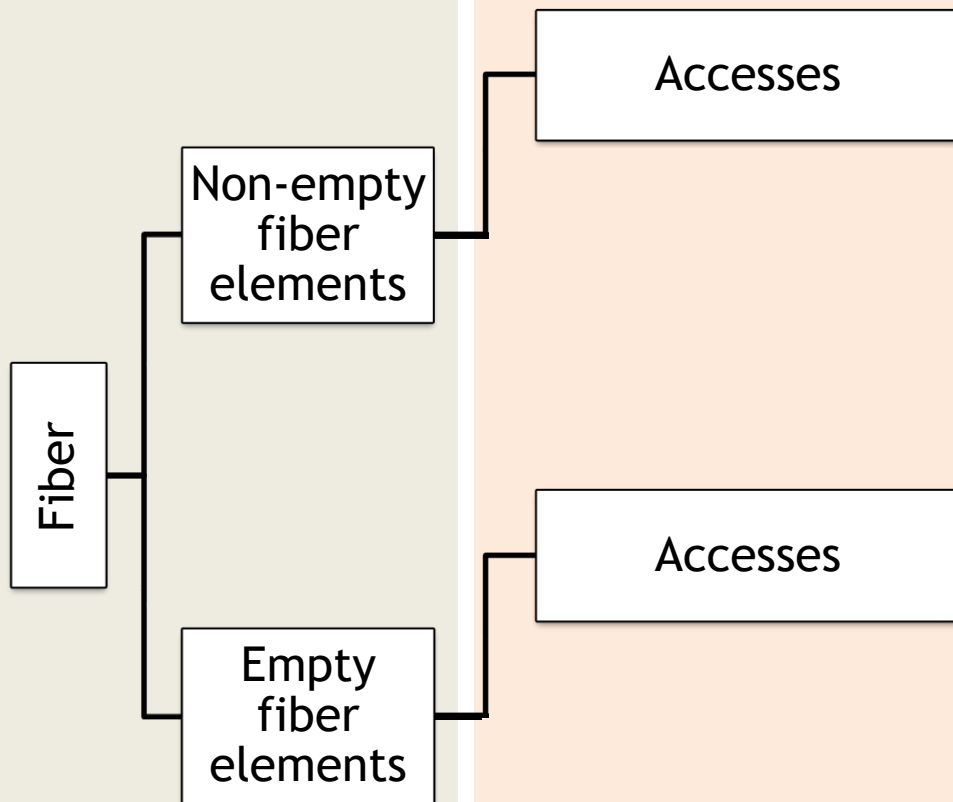


Total: 6 actual accesses, 6 cycles

Sparse Optimization Features Reduces Actual Accesses

Deterministic based
on the statistical
occupancy of fiber

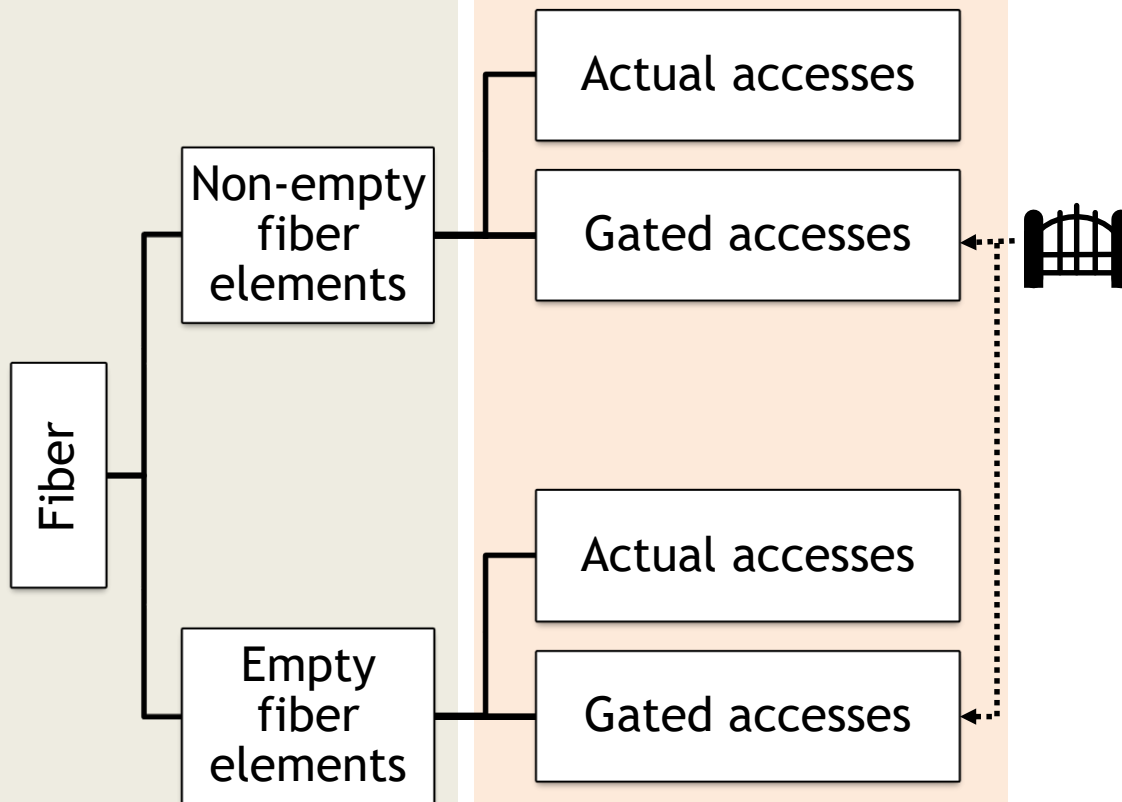
Dependent on sparse
optimization features
applied



Gating Leads to Gated Accesses

Deterministic based
on the statistical
occupancy of fiber

Dependent on sparse
optimization features
applied

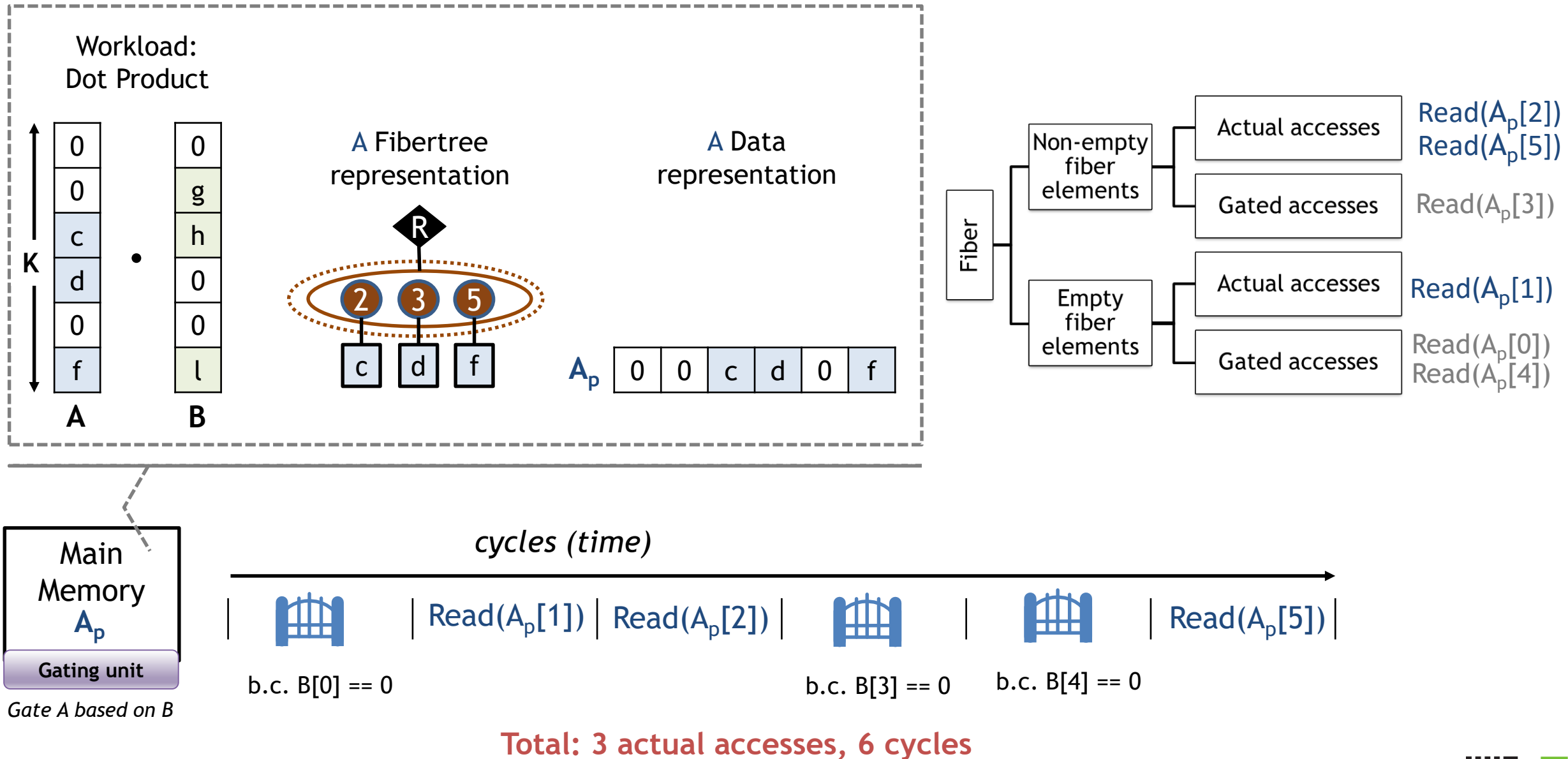


Gating:

Explicit energy saving of access to the payload* of one element of a fiber based on the emptiness of an element of another fiber

**Note that since the "payload" of an element of a fiber may be a whole fiber (or tree of fibers) more than one accesses can be optimized*

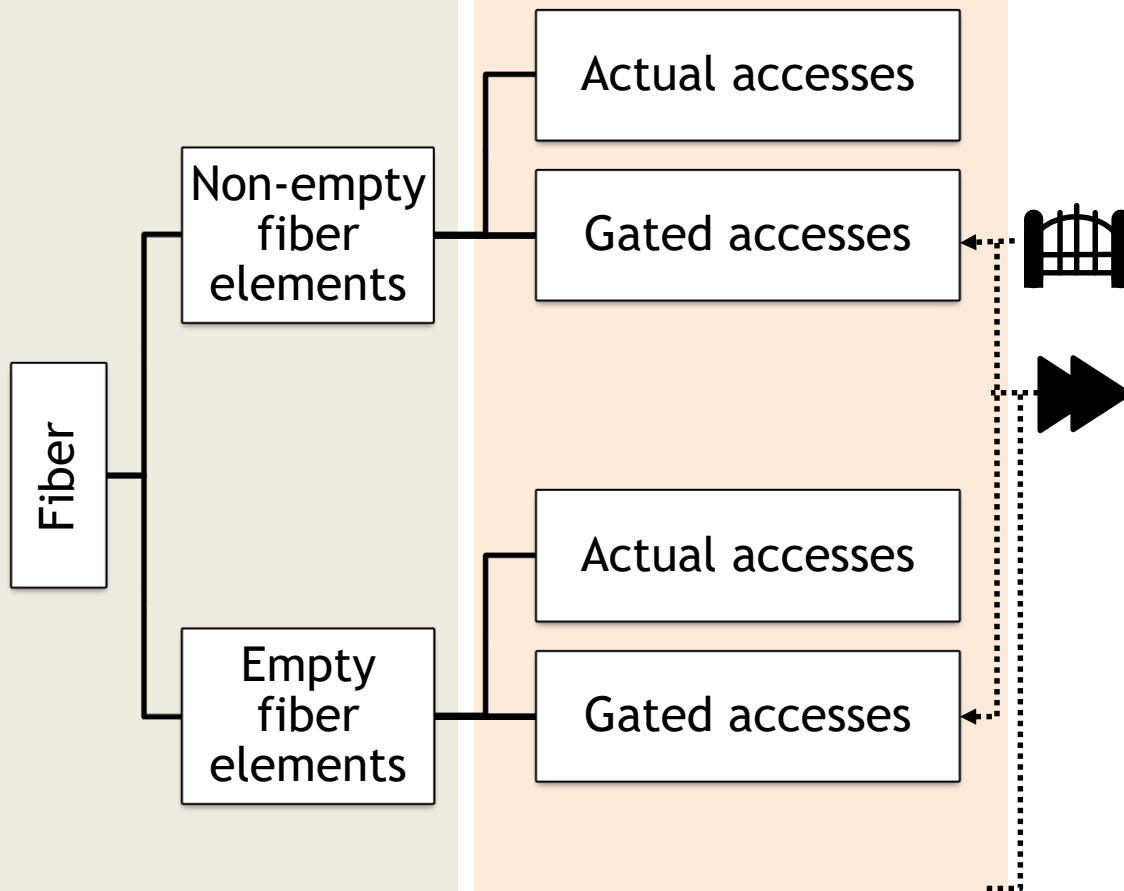
Zero-Gated A Tensor Accesses in A Dot Product Workload



Skipping Leads to Skipped Accesses

Deterministic based
on the statistical
occupancy of fiber

Dependent on sparse
optimization features
applied



Gating:

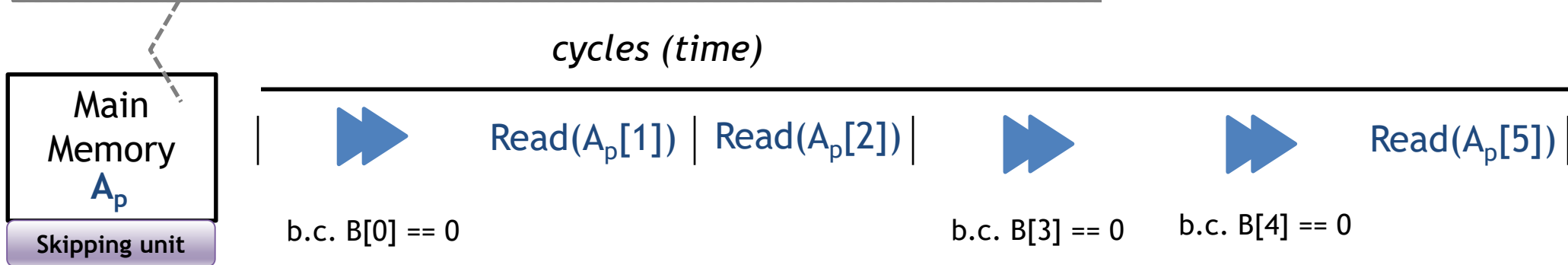
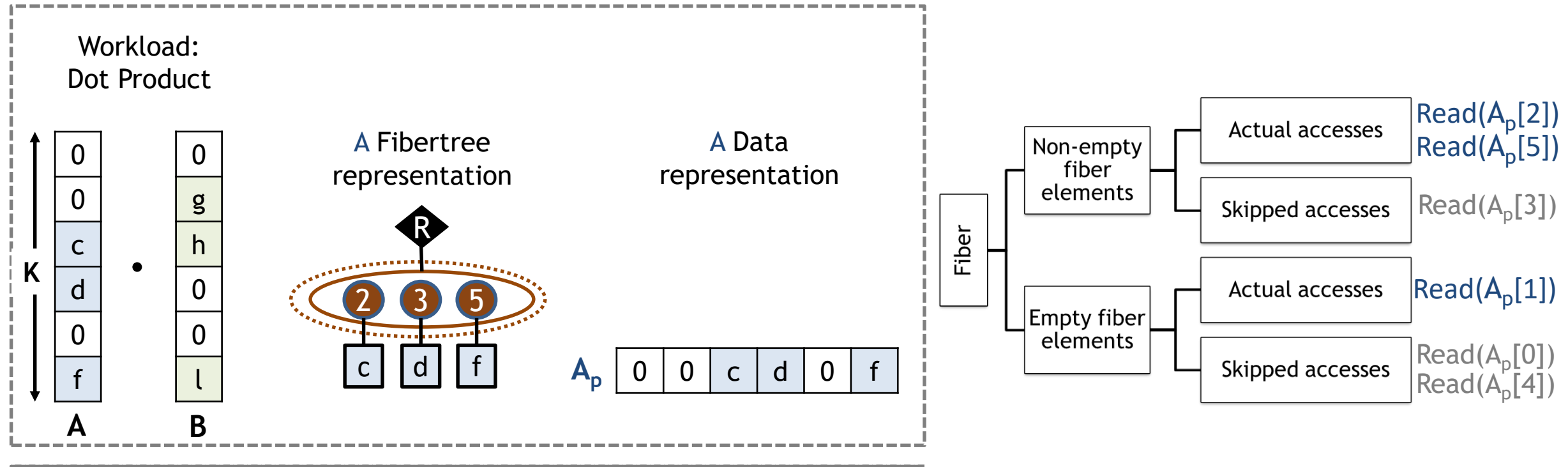
Explicit energy saving of access to the payload* of one element of a fiber based on the emptiness of an element of another fiber

Skipping:

Explicit skipping over access to the payload* of one element of a fiber based on the emptiness of an element of another fiber

**Note that since the "payload" of an element of a fiber may be a whole fiber (or tree of fibers) more than one accesses can be optimized*

Zero-Skipped A Tensor Accesses in A Dot Product Workload



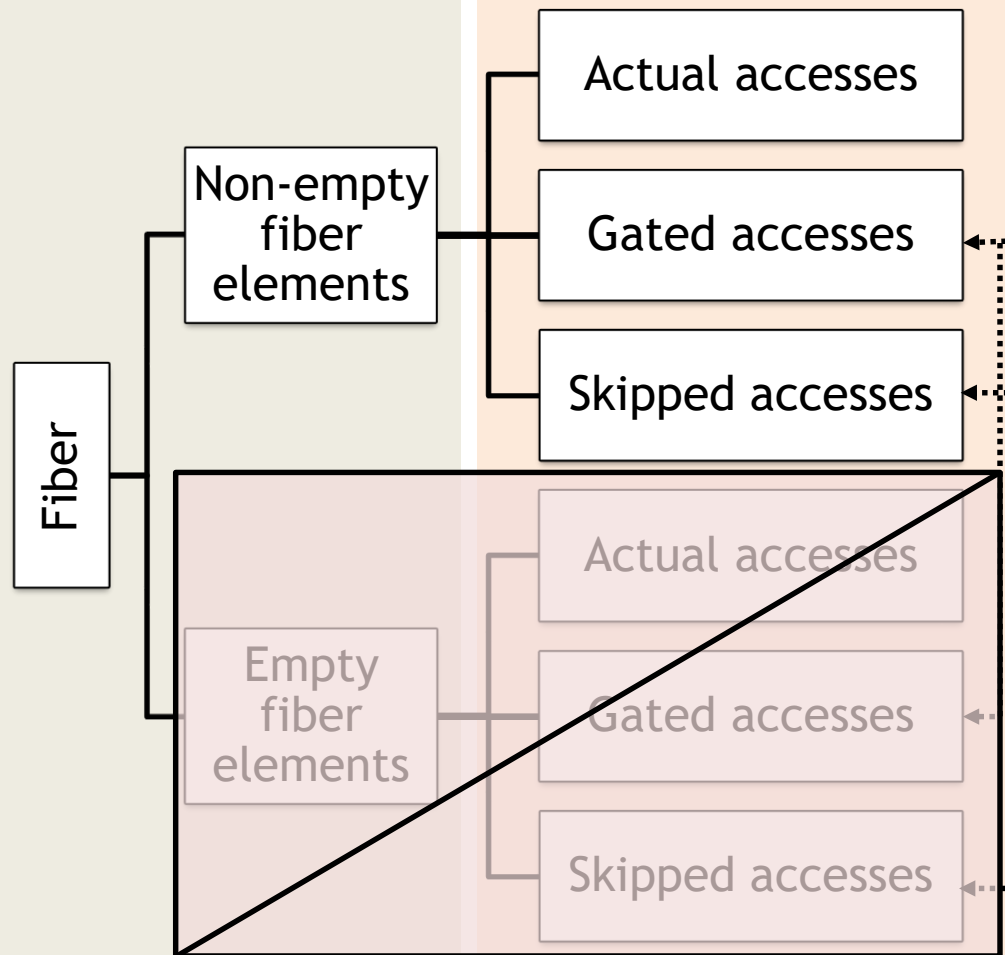
Skip A based on B

Total: 3 actual accesses, 3 cycles

Compression Eliminates Accesses to Empty Elements

Deterministic based
on the statistical
occupancy of fiber

Dependent on sparse
optimization features
applied



Gating:

Explicit energy saving of access to the payload* of one element of a fiber based on the emptiness of an element of another fiber

Skipping:

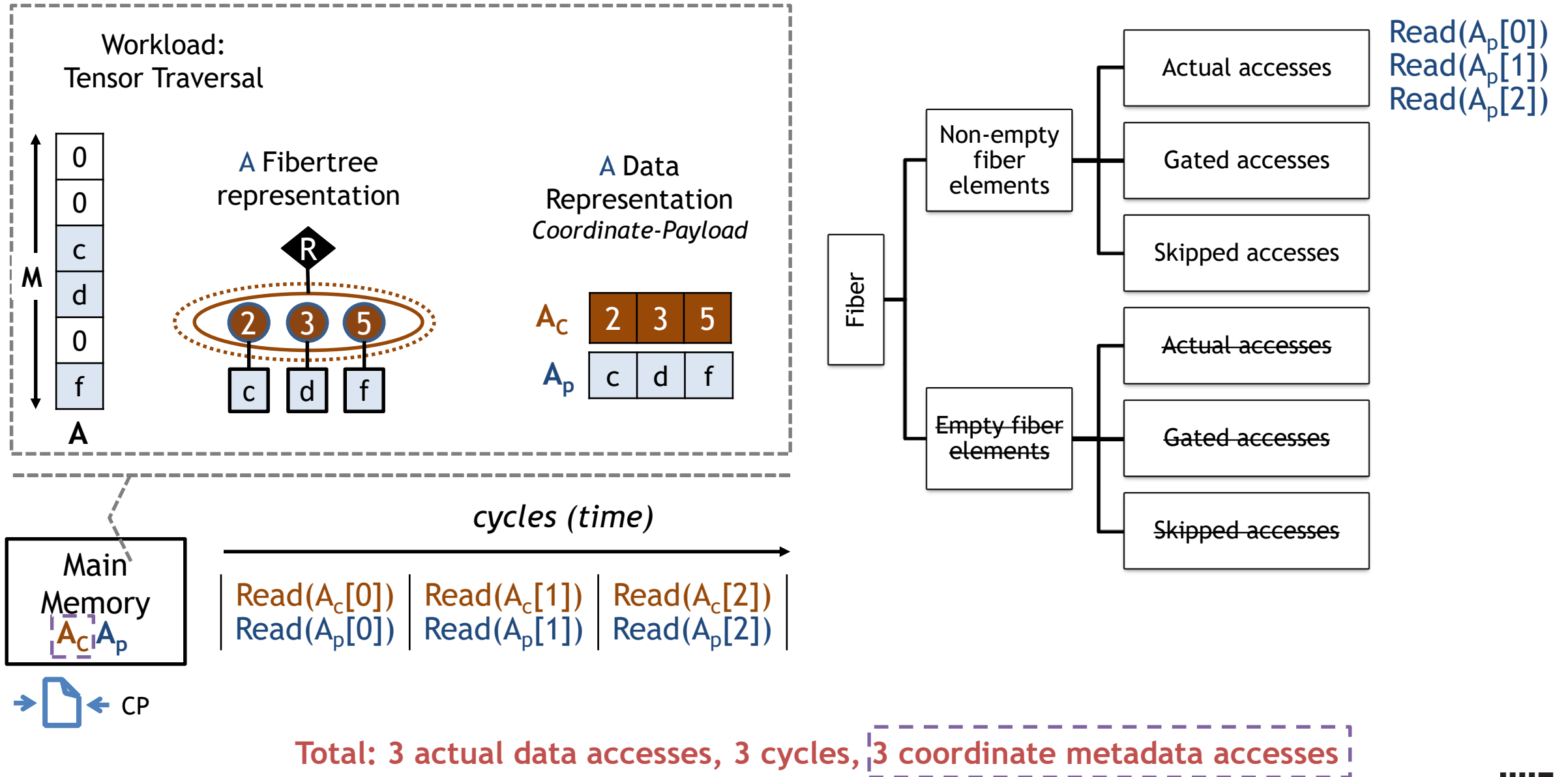
Explicit skipping over access to the payload* of one element of a fiber based on the emptiness of an element of another fiber

Format:

Choose data representation formats to save storage space and/or allow better realization of gating and skipping

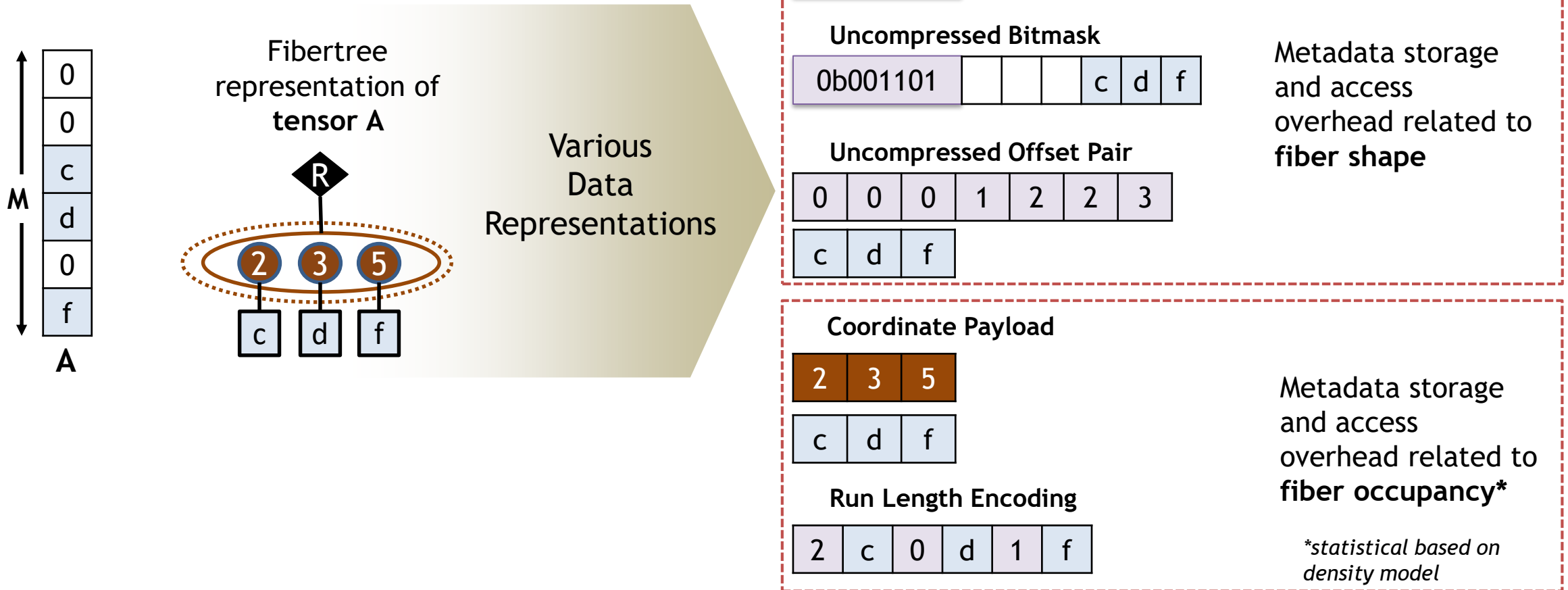
**Note that since the "payload" of an element of a fiber may be a whole fiber (or tree of fibers) more than one accesses can be optimized*

A Tensor Traversal with Coordinate Payload Format



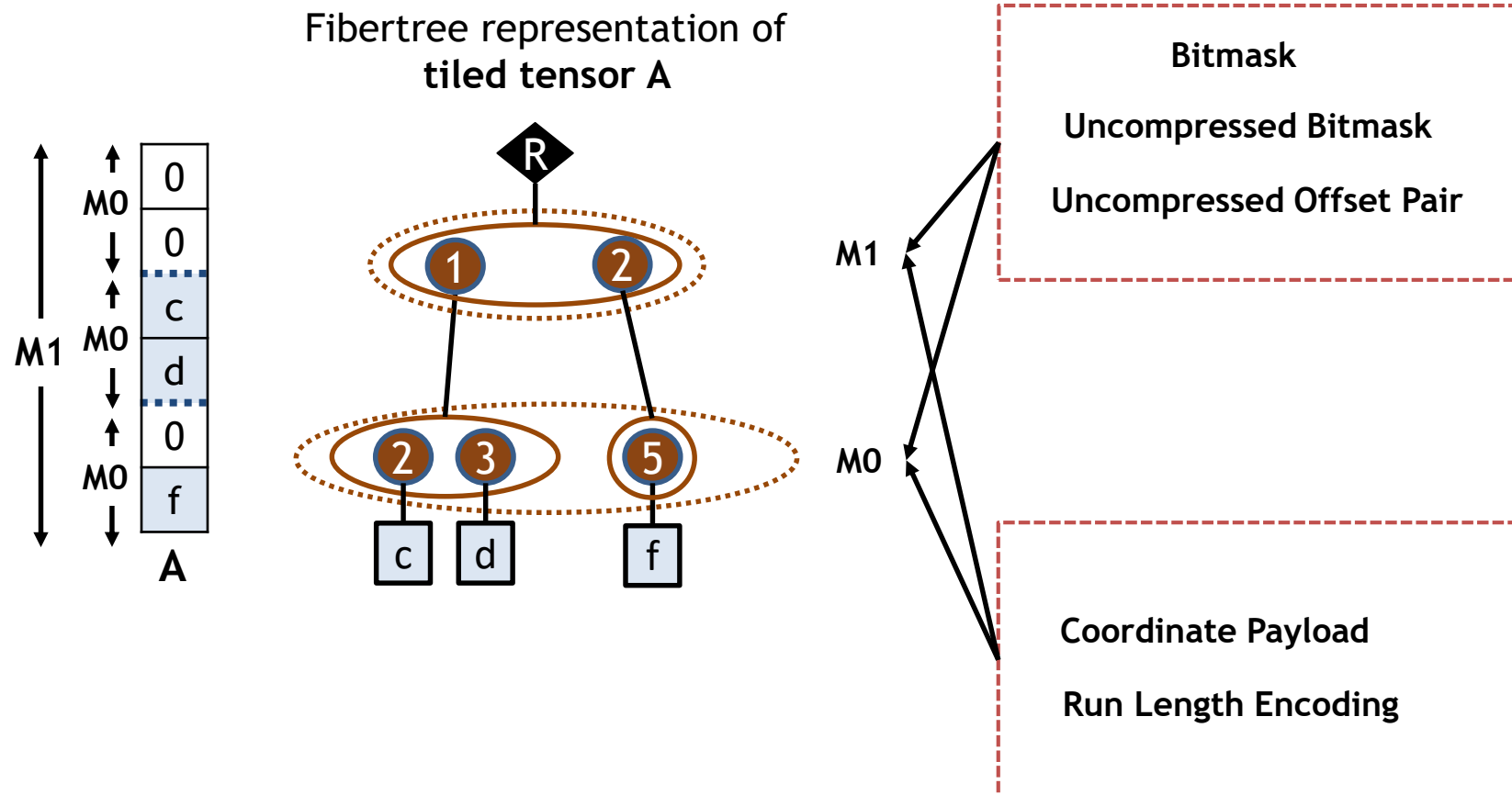
Format Choice Leads to Metadata Overhead

Metadata that identifies the locations of zeros is necessary



Multi-Rank Metadata Overhead

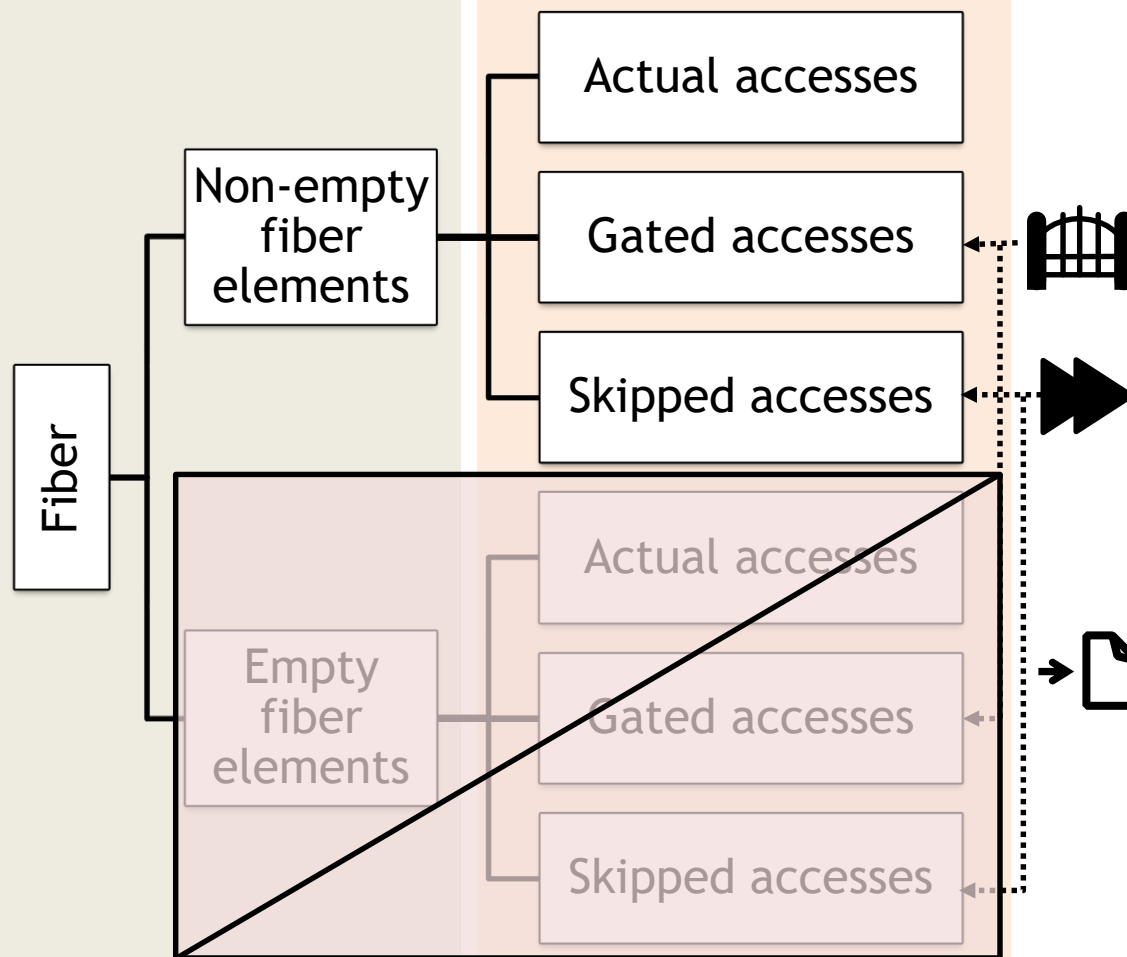
Per-Rank Occupancy and Access Analysis Allows Modeling of Arbitrary Compression Format



Impact Defined by Fibers in Different Tensors

Deterministic based
on the statistical
occupancy of fiber

Dependent on sparse
optimization features
applied



Gating:

Explicit energy saving of access to the payload* of one element of a fiber based on the emptiness of an element of another fiber

Dependent on
another tensor's
density

Skipping:

Explicit skipping over access to the payload* of one element of a fiber based on the emptiness of an element of another fiber

Format:

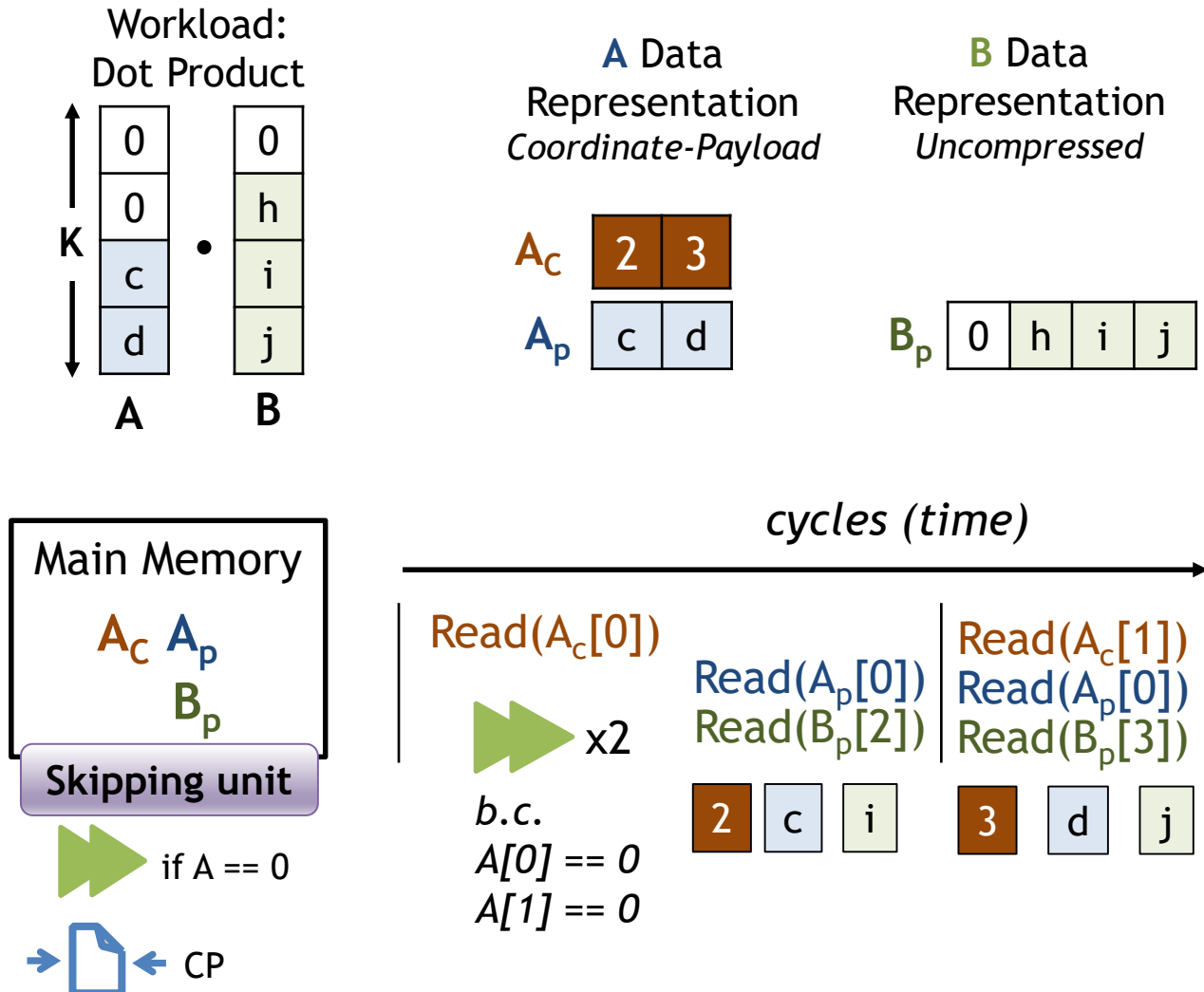
Choose data representation formats to save storage space and/or allow better realization of gating and skipping

Dependent on the
tensor's own
density

**Note that since the "payload" of an element of a fiber may be a whole fiber (or tree of fibers) more than one accesses can be optimized*

Interplay Between Different Sparse Optimization Features

Multiple sparse optimization features can be applied at the same time
As a result, the impact on required storage capacity and storage accesses aggregates

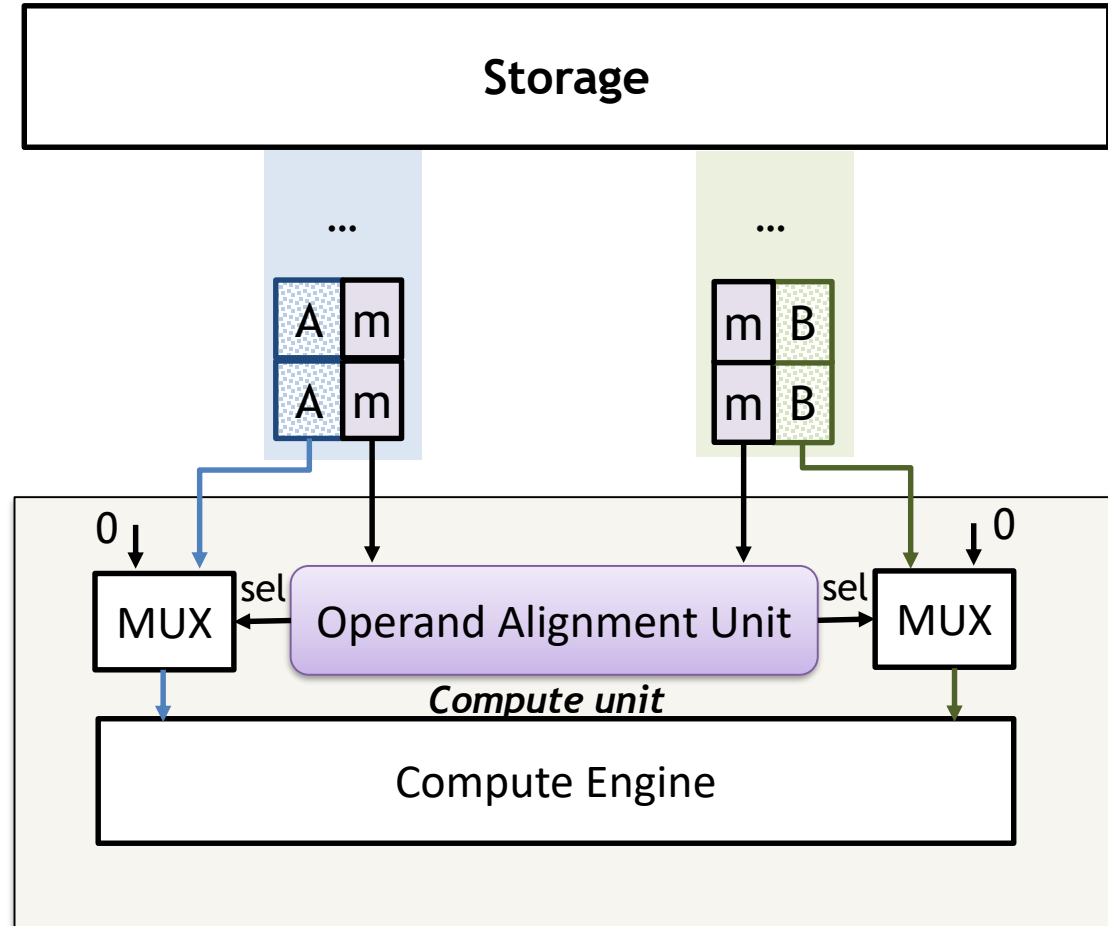


```
for (a_c, a_p) in A:  
    Z[a_c] += a_p * B_p[a_c]
```

Each A_c value is 2 bits
Each A_p value is 8 bits
Each B_p value is 8 bits

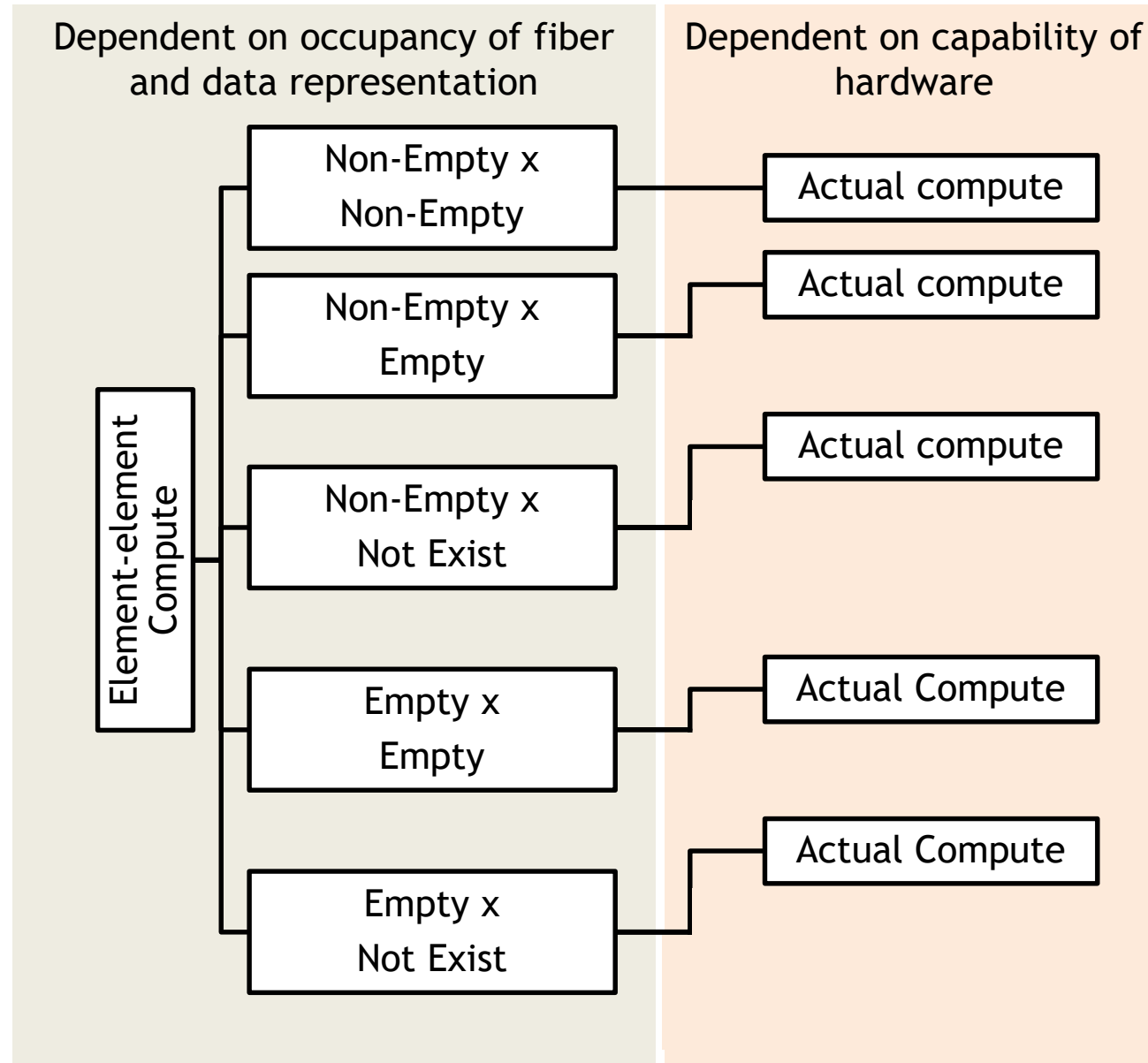
- Processing time reduced by 2x
- Hardware capacity requirement reduced by 1.23x
- Number of payload storage accesses reduced by 2x
- Incurs 2 extra metadata storage access overhead

Baseline Compute Unit Hardware Setup



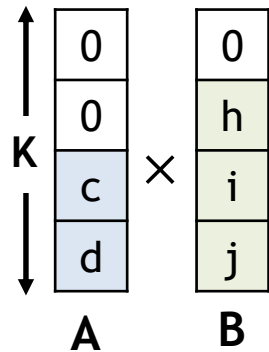
Operand alignment unit checks operand metadata and decides whether the incoming operands correspond to each other

Sparse Optimization Features Lead to Different Types of Computes

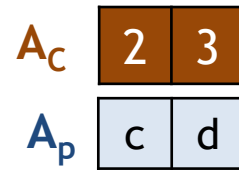


Baseline Compute Unit Working on Dot Product

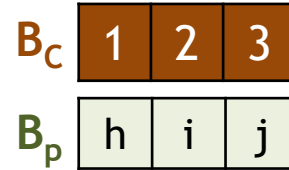
Workload:
Dot Product



A Data
Representation
Coordinate-Payload

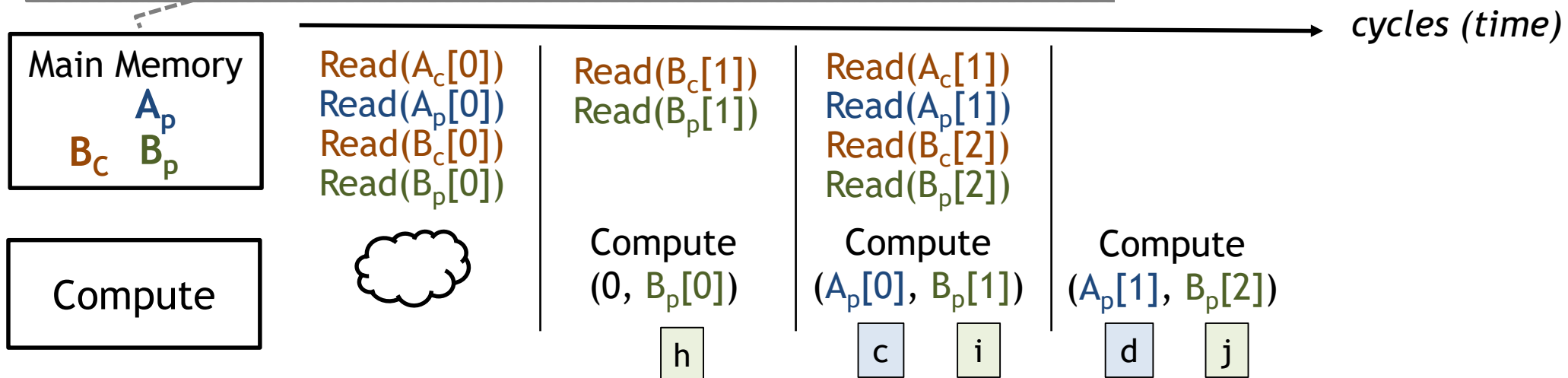


B Data
Representation
Coordinate-Payload

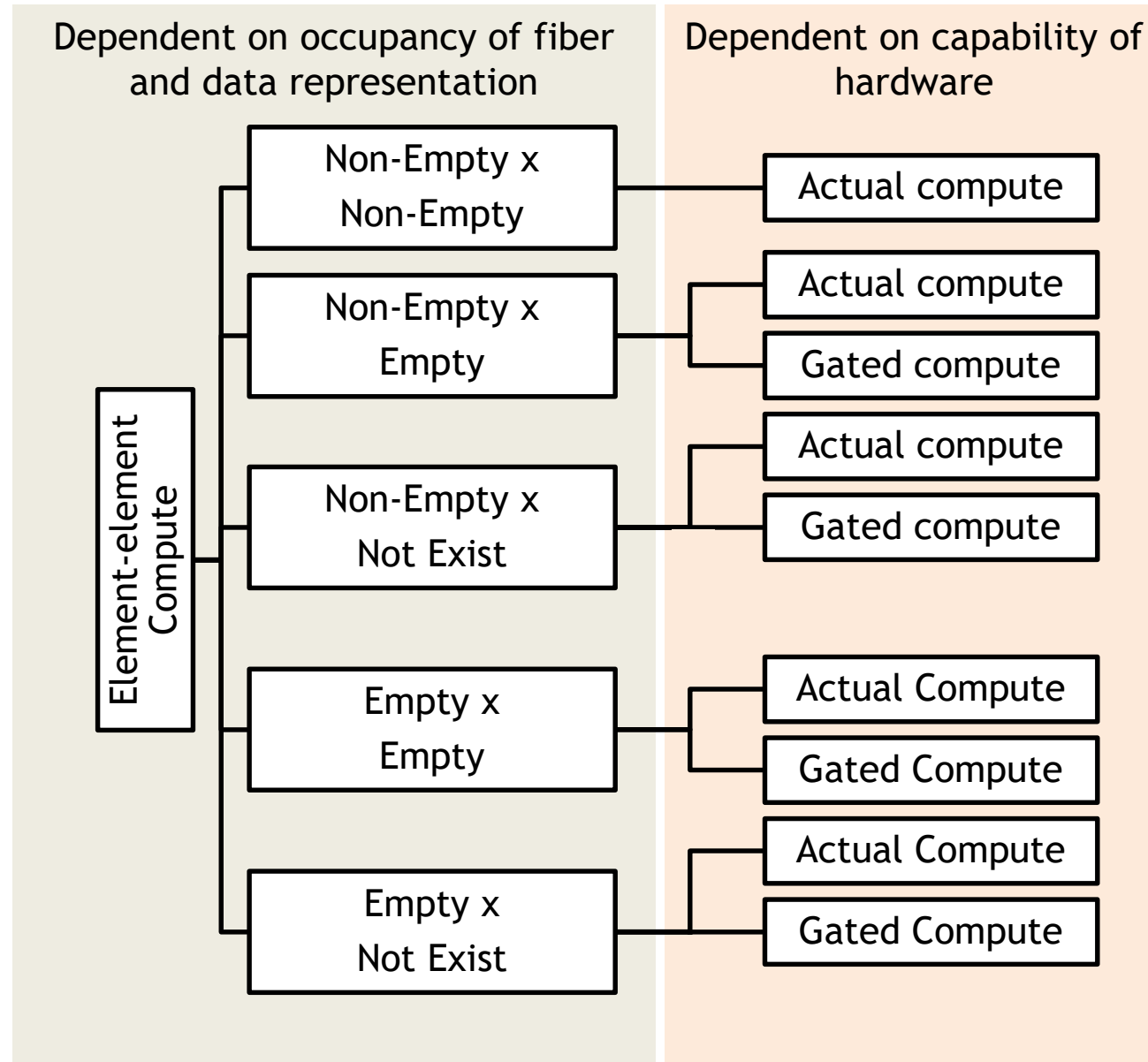


K : contracted dimension

It is important to align the contracted dimension to perform a valid compute



Sparse Optimization Features Lead to Different Types of Computes

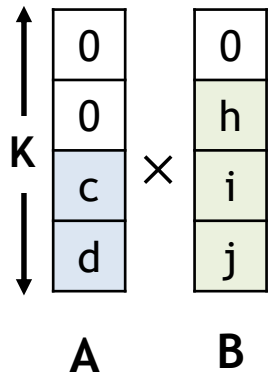


Gating:

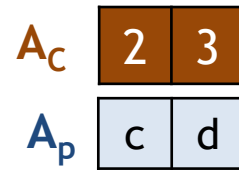
Explicit energy saving of compute when one of the payloads of operand elements is empty (i.e., compute engine recognizing zero operands)

Gated Compute Unit Working on Dot Product

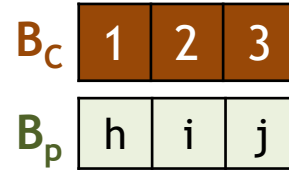
Workload:
Dot Product



A Data
Representation
Coordinate-Payload

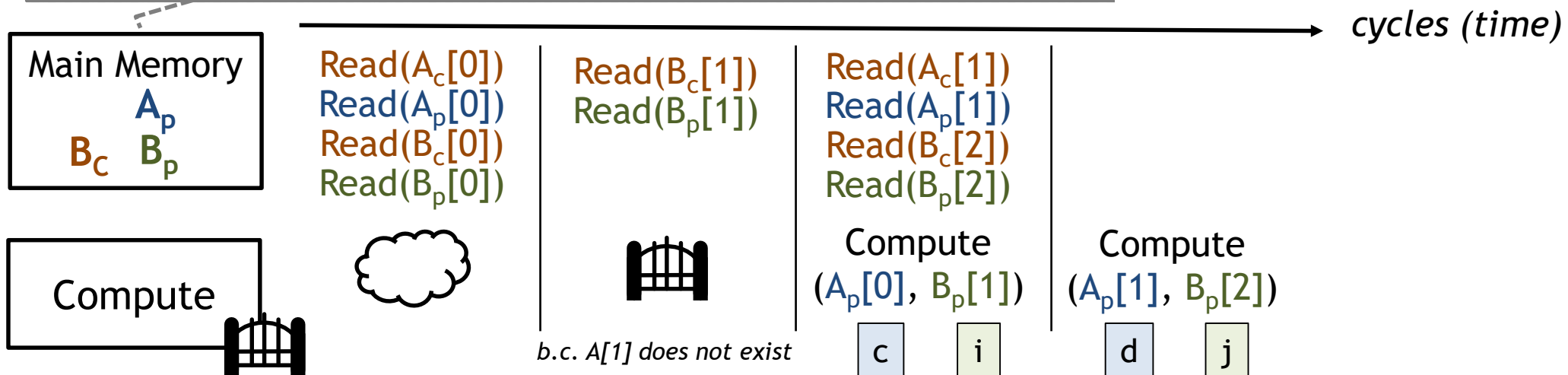


B Data
Representation
Coordinate-Payload

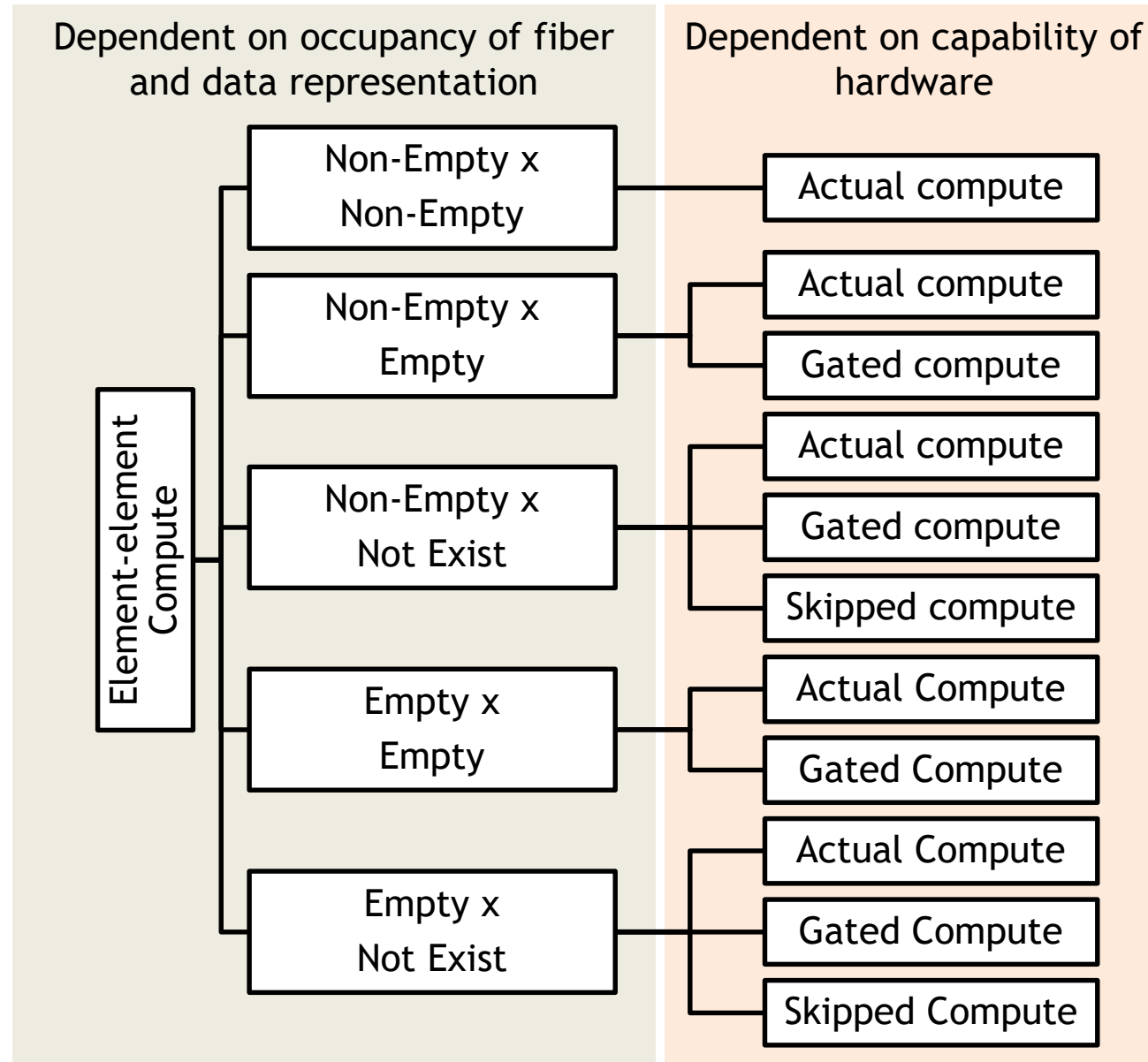


K: contracted dimension

It is important to align the contracted dimension to perform a valid compute



Sparse Optimization Features Lead to Different Types of Computes



Gating:

Explicit energy saving of compute when one of the payloads of operand elements is empty (i.e., compute engine recognizing zero operands)



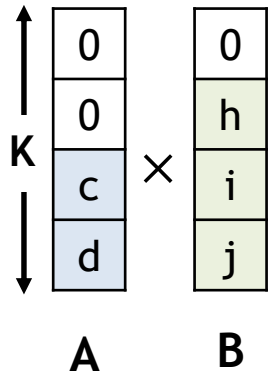
Skipping:

Explicit skipping over a compute when one of the payloads of operand elements does not exist (i.e., look-up based operand alignment)

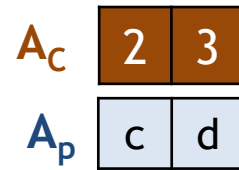
Note: skipping cannot skip over empty elements

Skipped Compute Unit Working on Dot Product

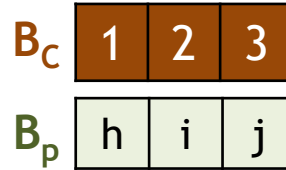
Workload:
Dot Product



A Data
Representation
Coordinate-Payload



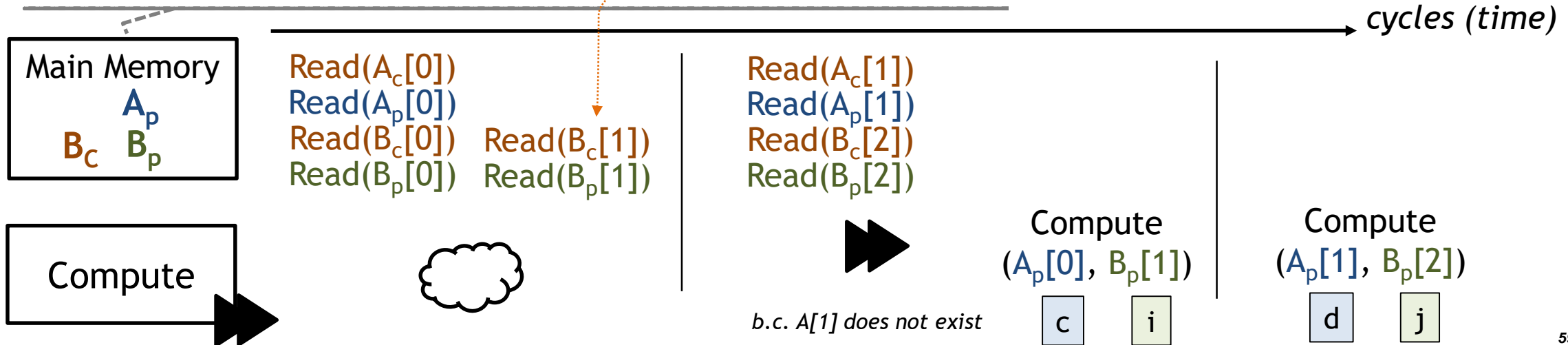
B Data
Representation
Coordinate-Payload



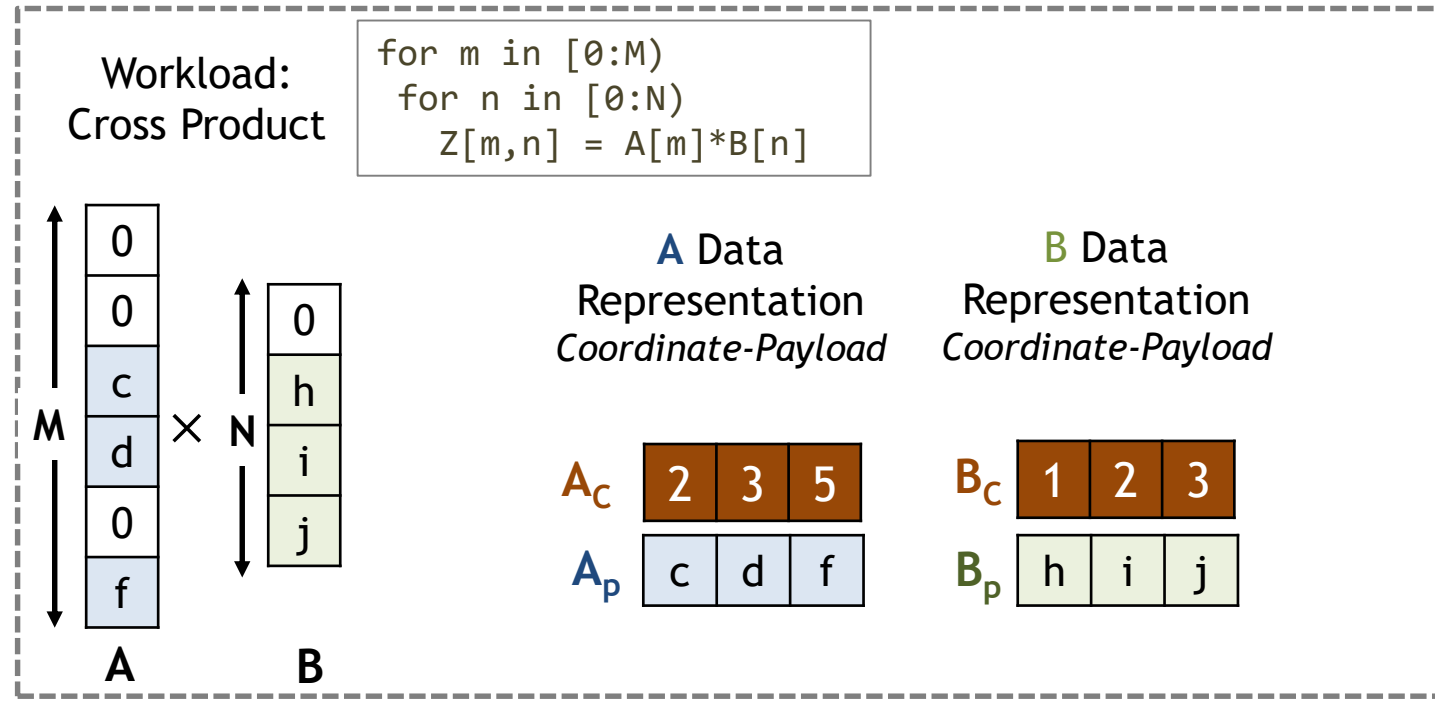
K: contracted dimension

It is important to align the contracted dimension to perform a valid compute

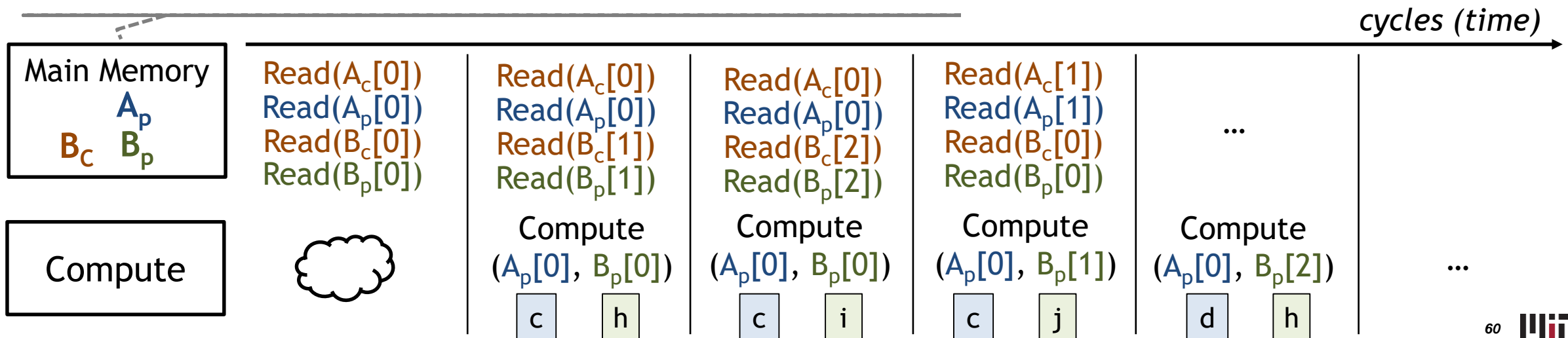
Assume we have enough bandwidth to read out two Bs to the compute unit in one cycle



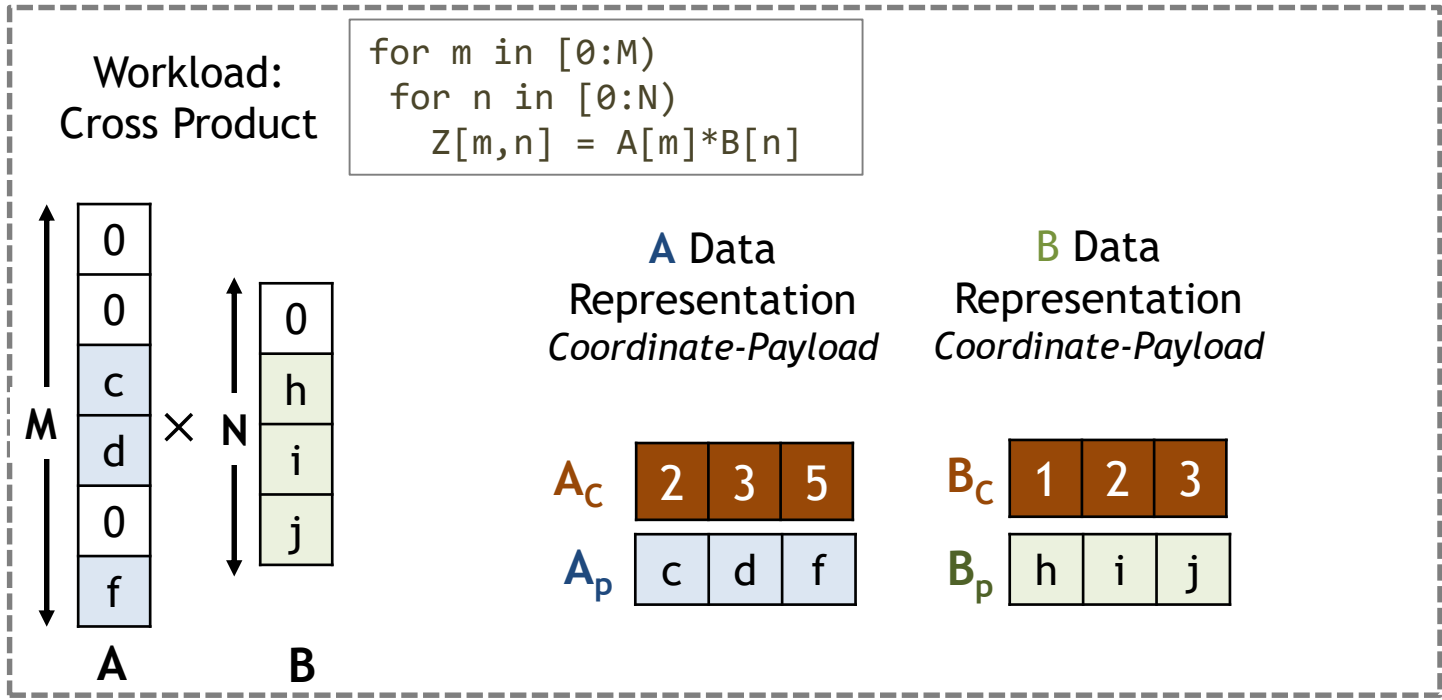
Baseline Compute Unit Working on Cross Product



There is no contracted dimension in a cross product, no alignment needed

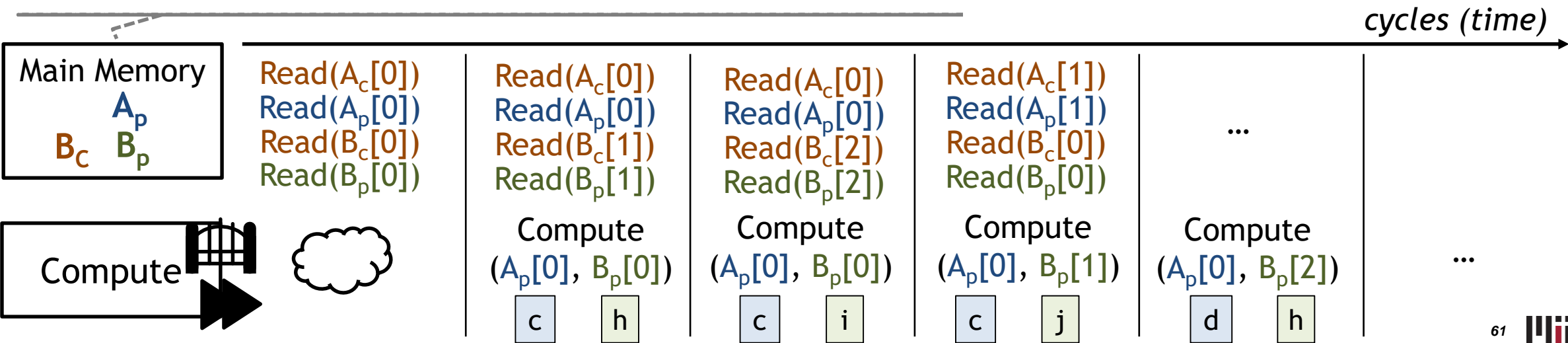


Interactions between Problem Spec and Opt. Features



There is no contracted dimension in a cross product, no alignment needed

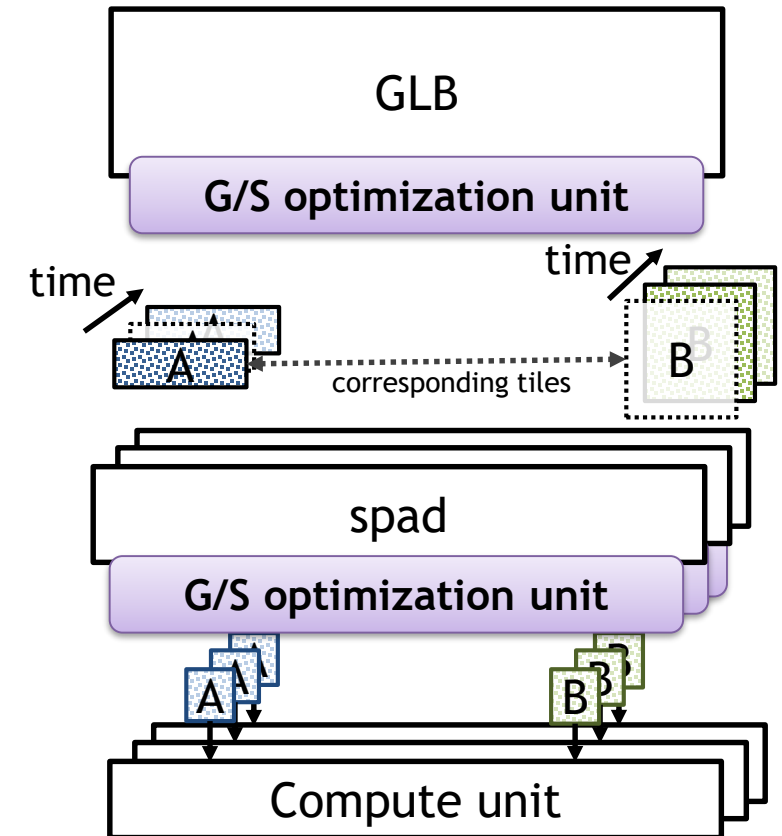
Gating/Skipping does not make a difference



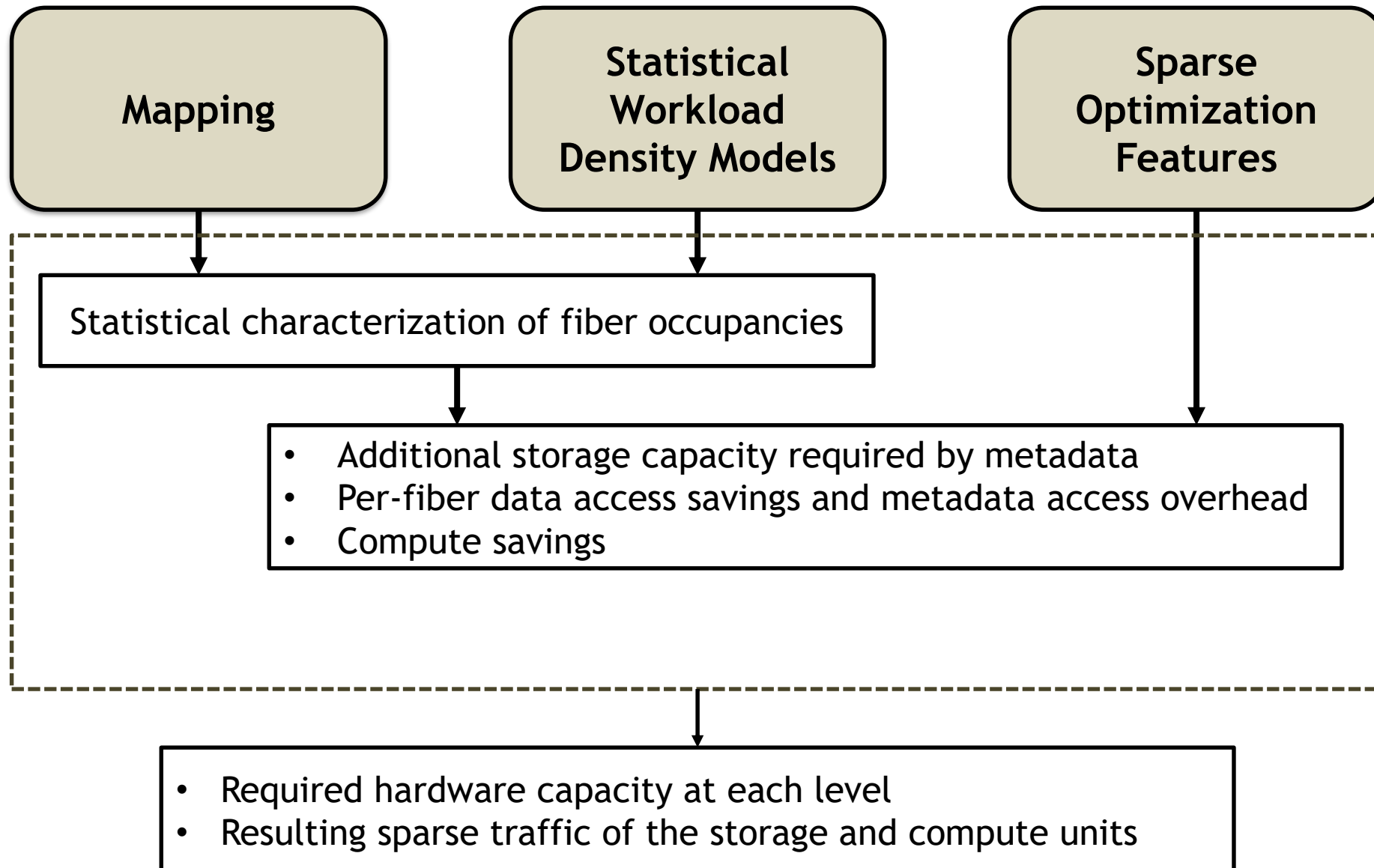
More Modeling Capabilities

- Zero-Gating and Zero-Skipping at intermediate storage levels
 - Propagation Impact to lower storage and compute levels
 - Choose gated/skipped tensor based on mapping
- Multi-rank compression formats
 - Interaction between compression formats and mapping
 - Compression with flattened ranks (important for deep neural network workloads)
 - Decompression at inner storage levels

More Realistic Multi-Level Architecture

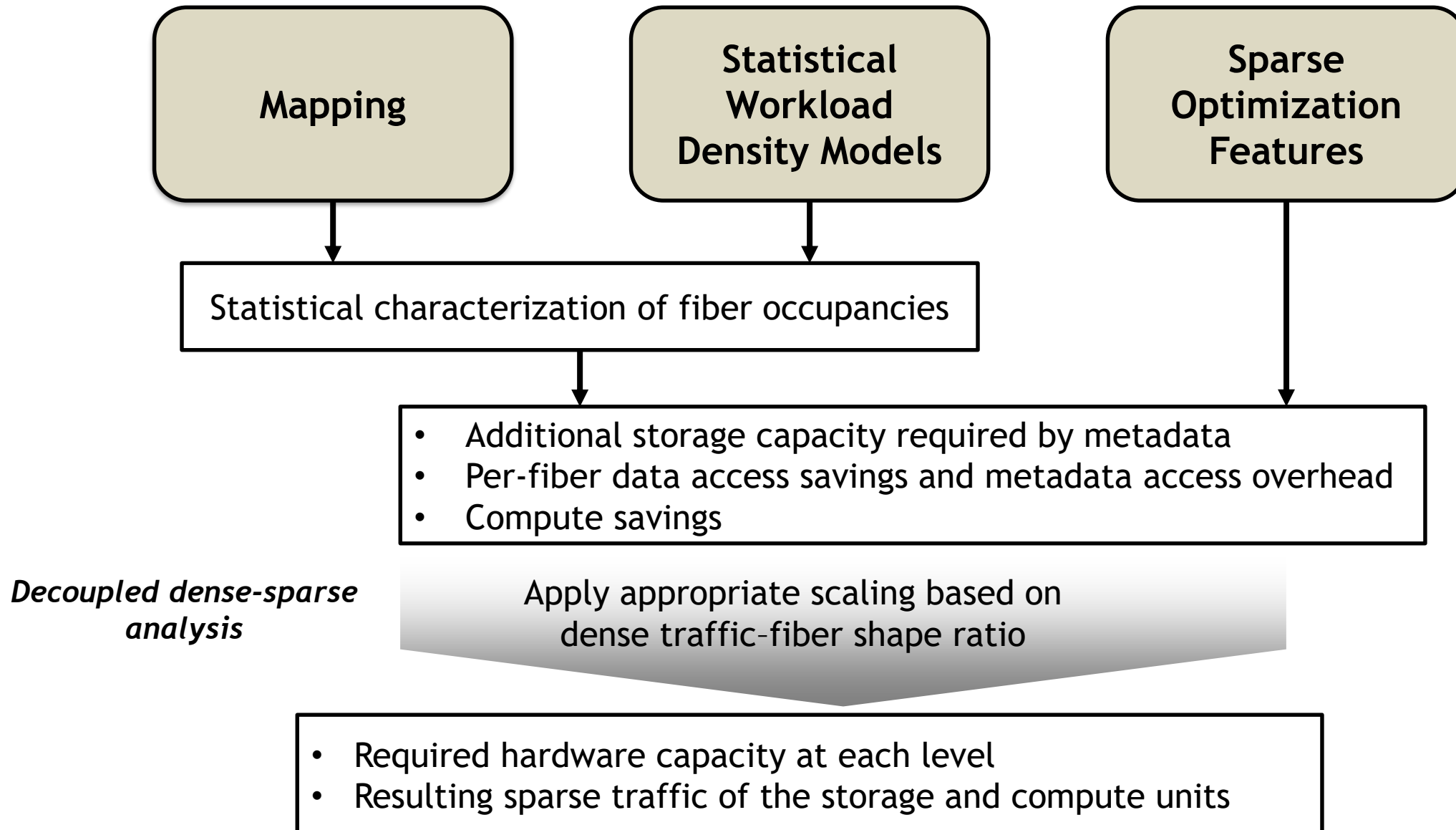


Specifications and Their Interactions



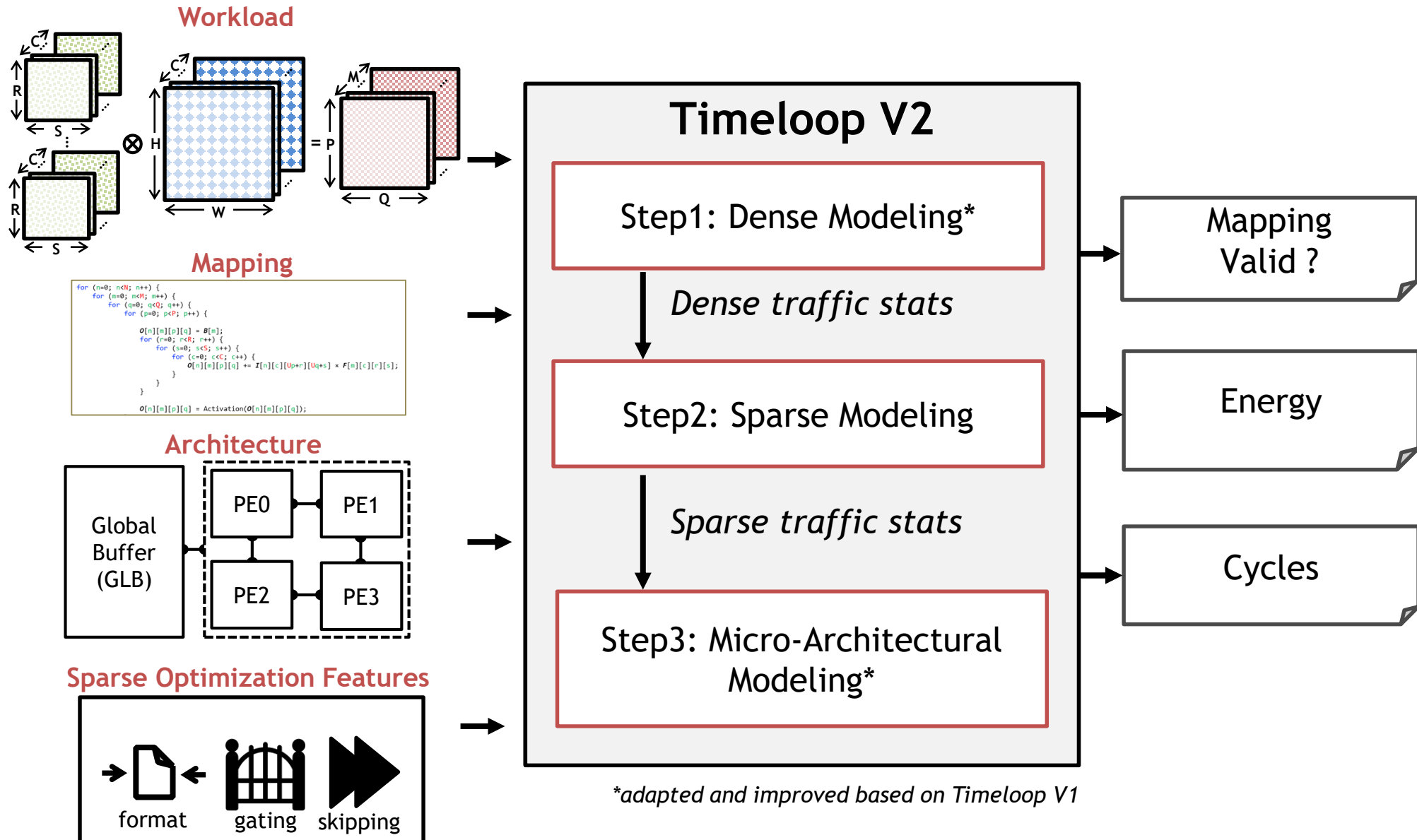
Interactions

Specifications and Their Interactions

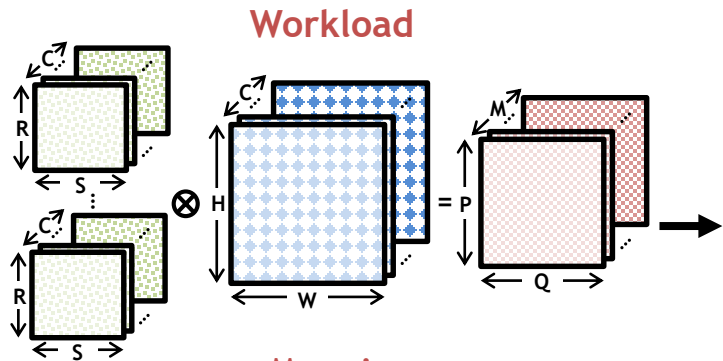


Timeloop V2 (a.k.a. Sparseloop) Infrastructure

Timeloop V2

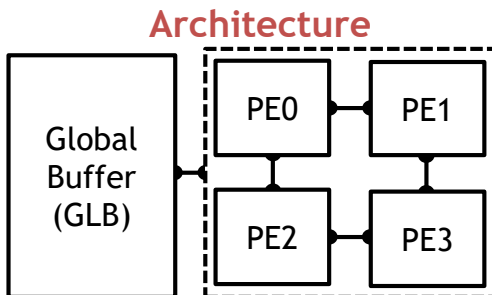


Timeloop V2 Inputs

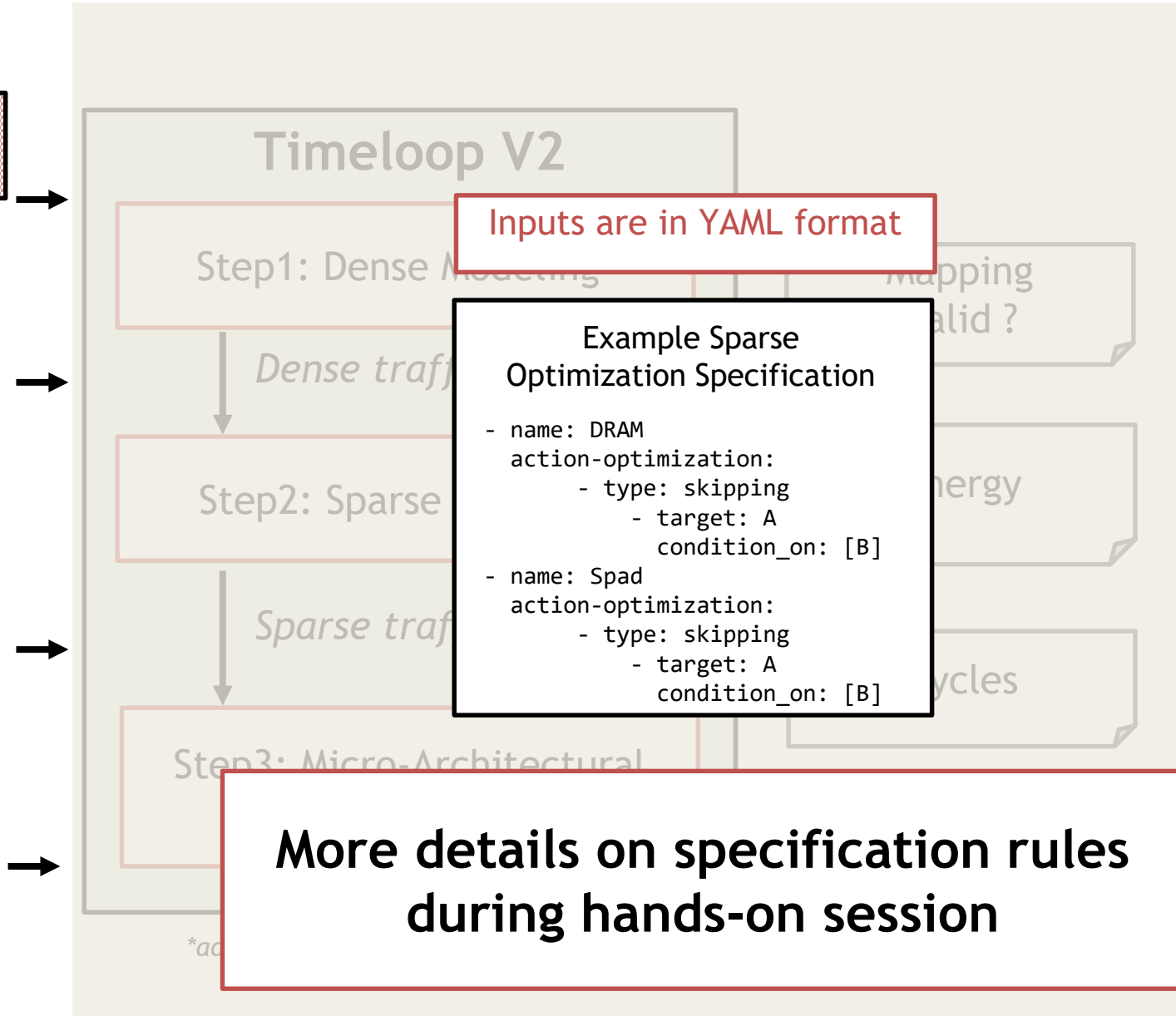
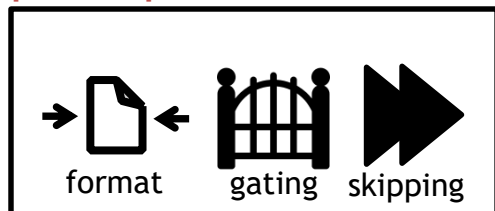


Mapping

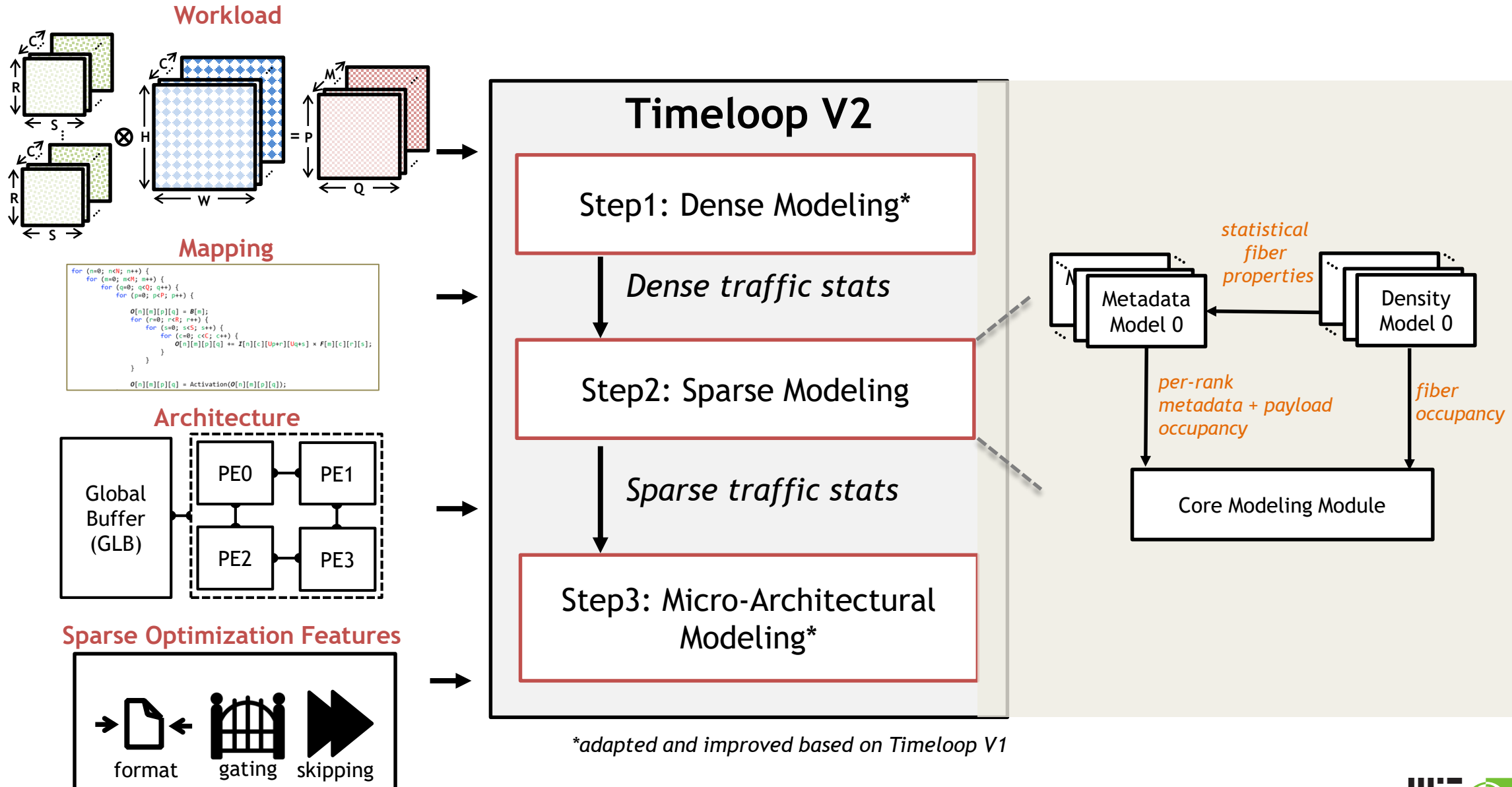
```
for (n=0; n<N; n++) {  
  for (m=0; m<M; m++) {  
    for (q=0; q<Q; q++) {  
      for (p=0; p<P; p++) {  
        O[n][m][p][q] = 0;  
        for (r=0; r<R; r++) {  
          for (c=0; c<C; c++) {  
            O[n][m][p][q] += I[n][c][Up[r][Uq++]] * F[m][c][r][+];  
          }  
        }  
      }  
    }  
  }  
}  
O[n][m][p][q] = Activation(O[n][m][p][q]);
```



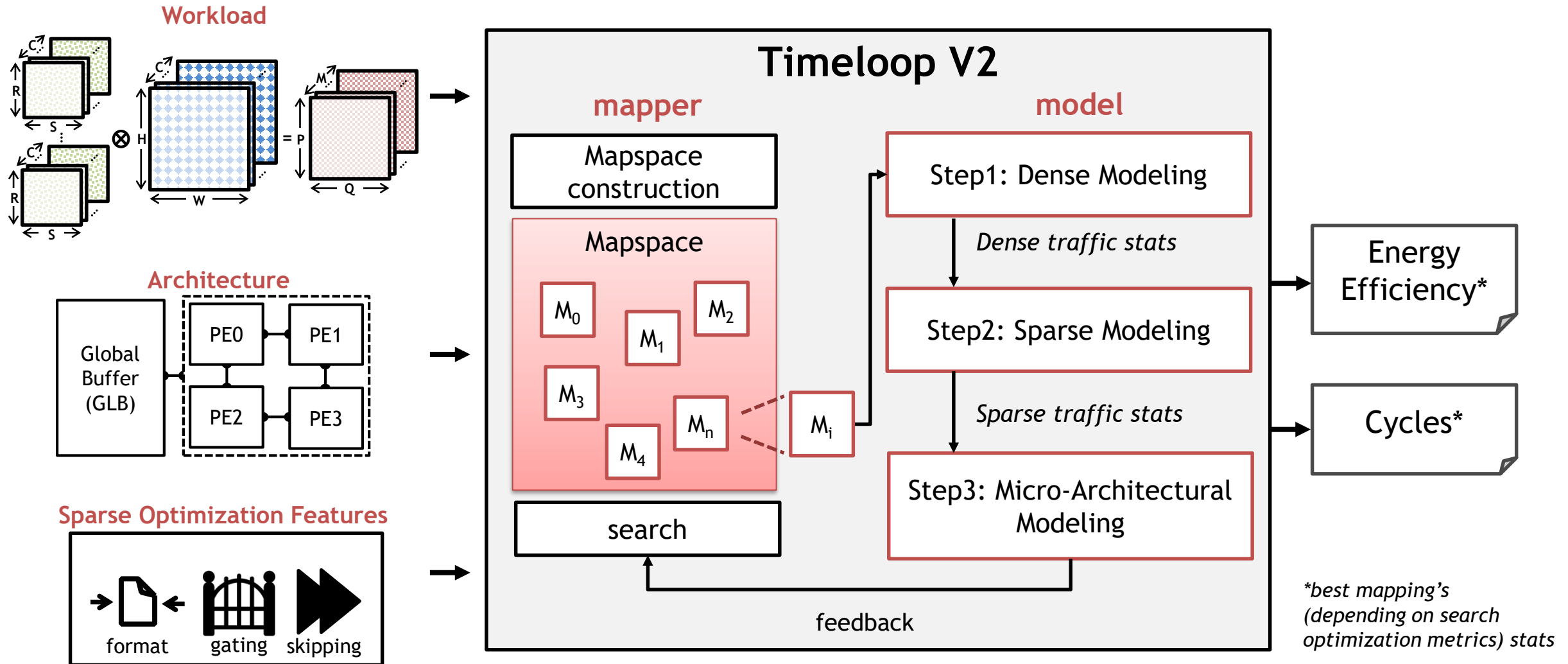
Sparse Optimization Features



Modularized Density and Format Models

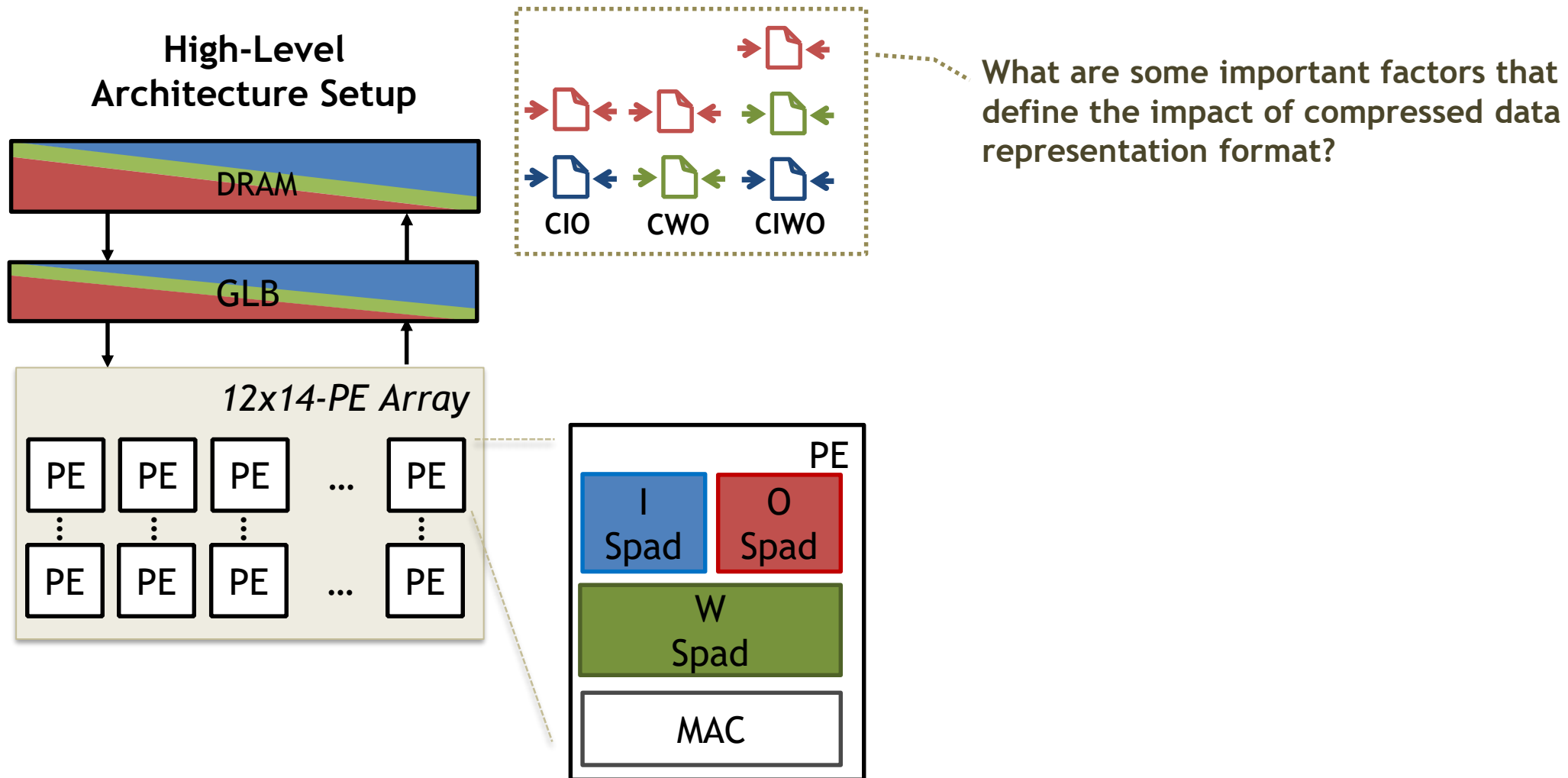


Timeloop V2 Mapspace Exploration



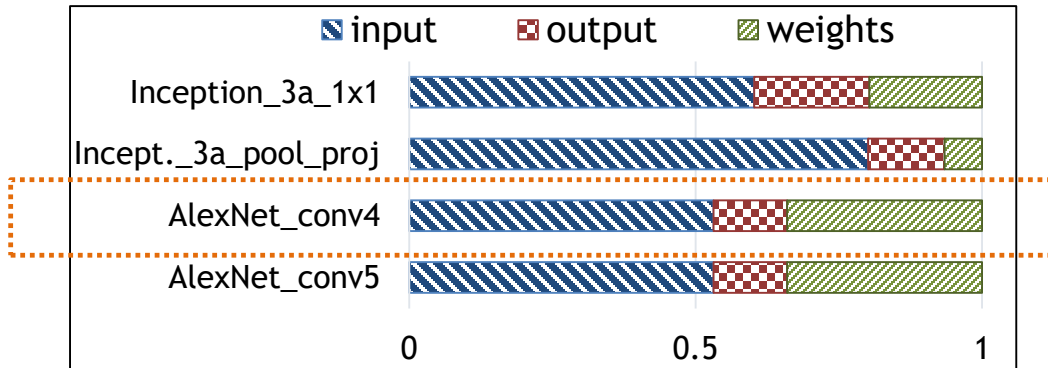
Case Studies

Explore different sparse optimization features



Uncompressed Traffic Breakdown vs. Compression Savings

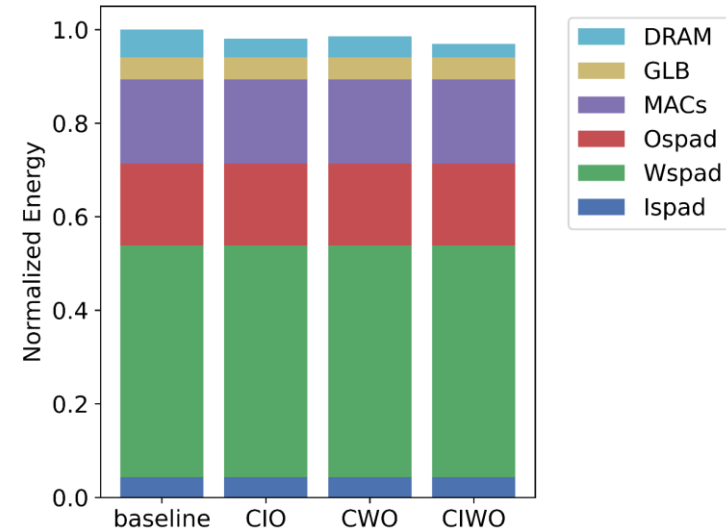
Uncompressed DRAM Traffic Breakdown



The tensor that dominates uncompressed traffic introduces more savings when compressed

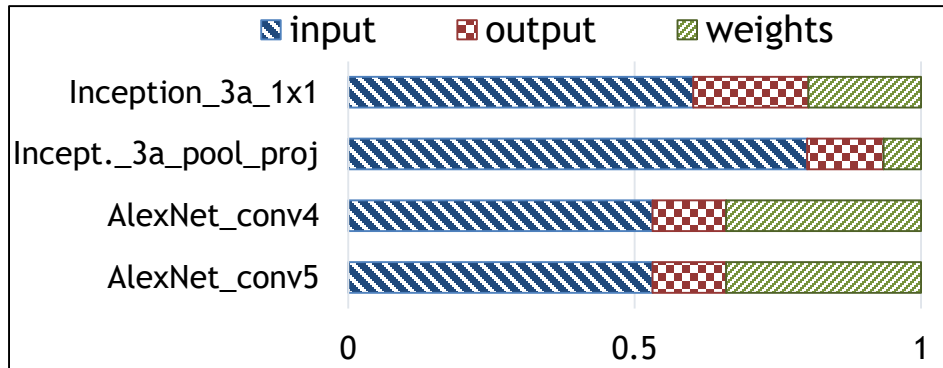
Is that true? **No**

AlexNet Conv4



Tensor Densities Play an Important Role

Uncompressed DRAM Traffic Breakdown



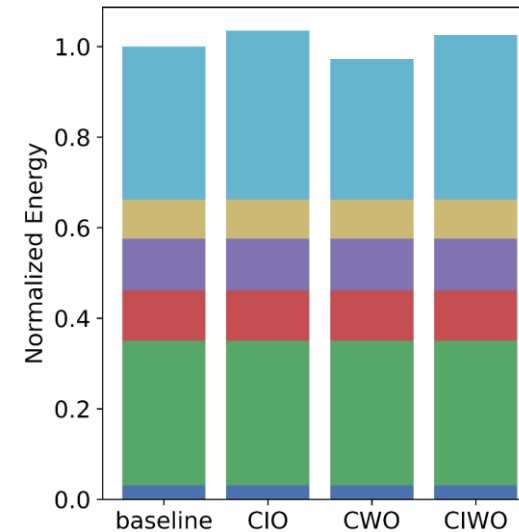
The tensor that dominates uncompressed traffic introduces more savings when compressed

Is that true? **No**

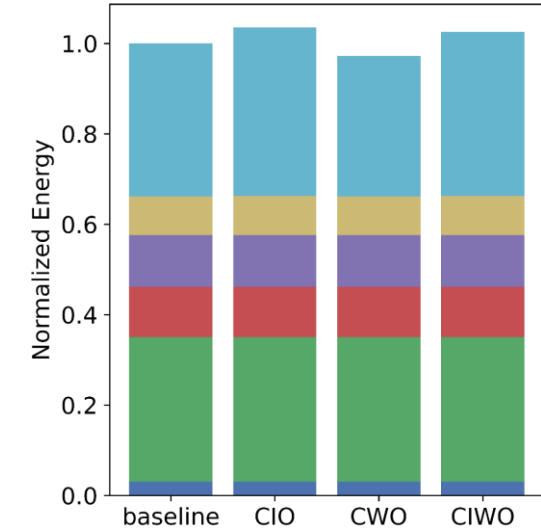
Layer Densities

Layer #	Inputs	Outputs	Weights
Inception_3a_1x1	0.71	0.66	0.37
Incept._3a_pool_proj	0.96	0.46	0.46
Alexnet_conv4	0.39	0.43	0.37
Alexnet_conv5	0.43	0.16	0.37

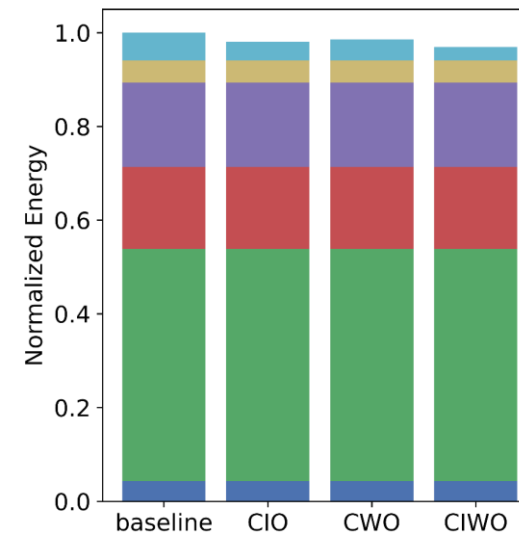
Inception_3a_1x1



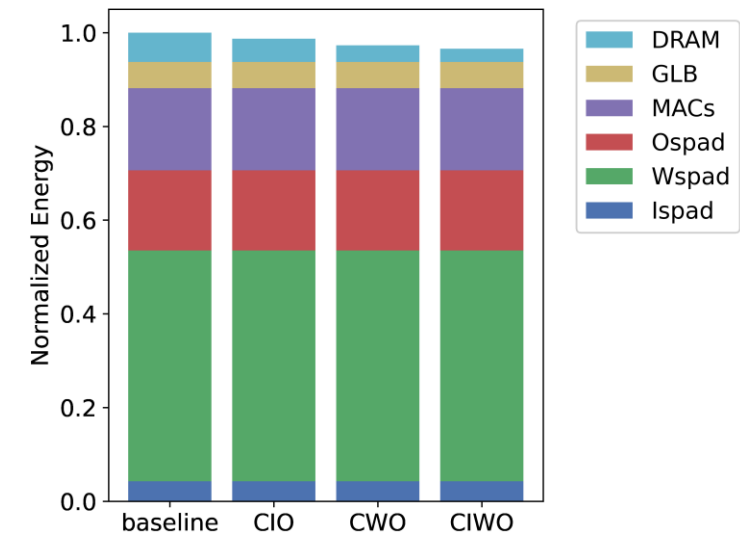
Inception_3a_pool_proj



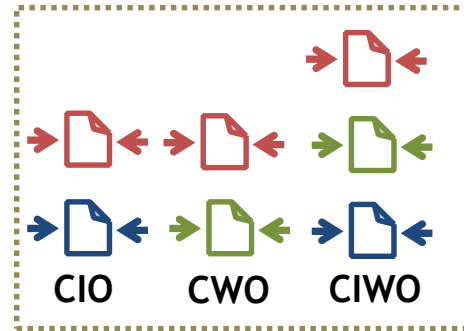
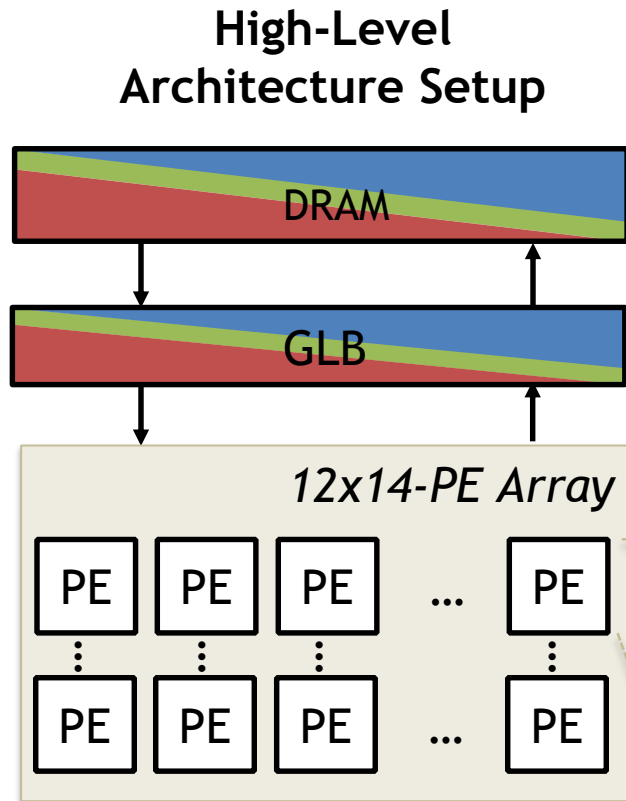
AlexNet Conv4



AlexNet Conv5

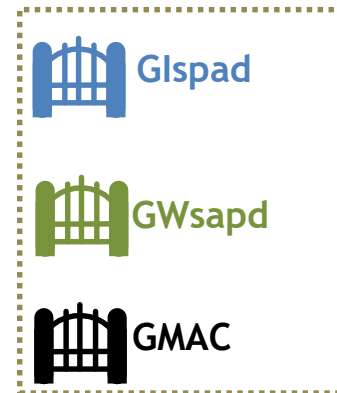
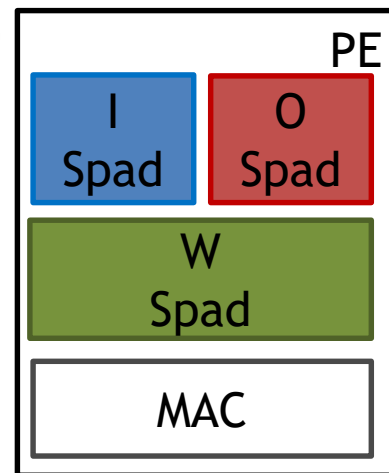


Explore different sparse optimization features



What are some important factors that define the impact of compressed data representation format?

- *Uncompressed traffic breakdown*
- *Tensor density*



What are some important factors that define the impact of gating on-chip?

Density vs. Gating Savings

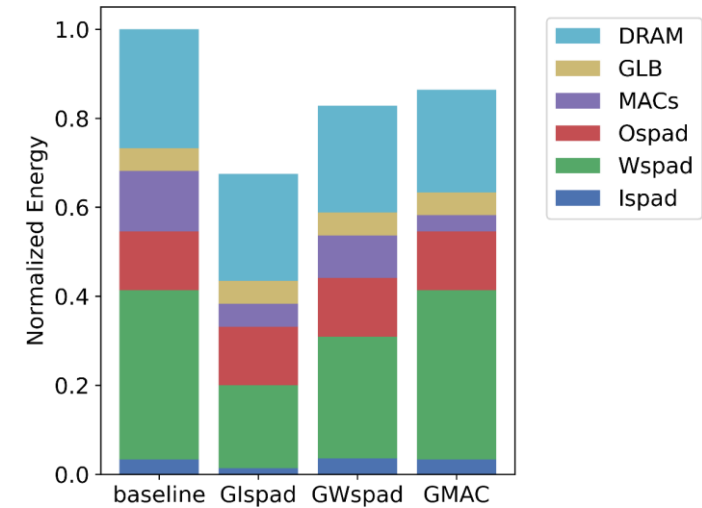
Layer Densities

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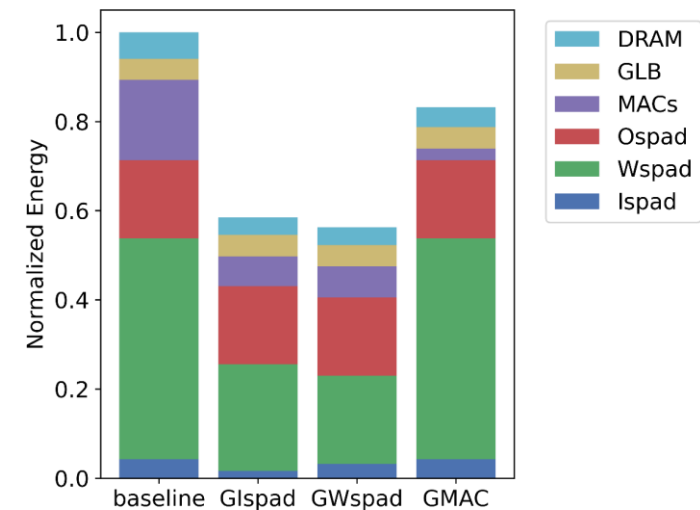
The tensor that has lower density should be the conditioned on tensor, i.e., it should have associated with metadata and allows the other tensor to be gated

Is that true? **No**

Inception_3a_1x1



AlexNet Conv4

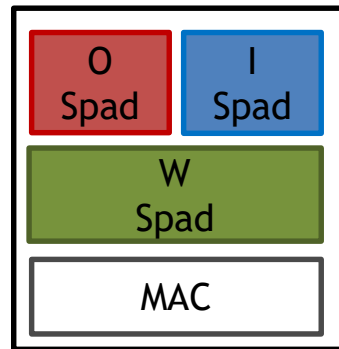


Hardware Attributes Plays an Important Role

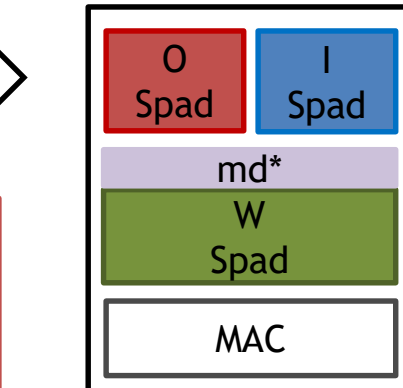
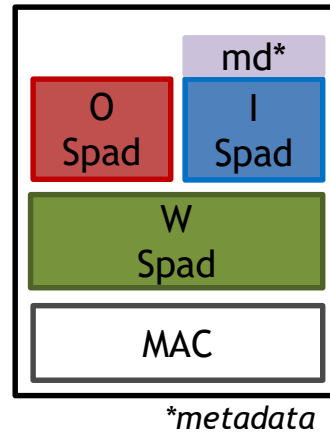
The tensor that has lower density should be the conditioned on tensor, i.e., it should have associated with metadata and allows the other tensor to be gated

Is that true? **No**

Original PE Architecture



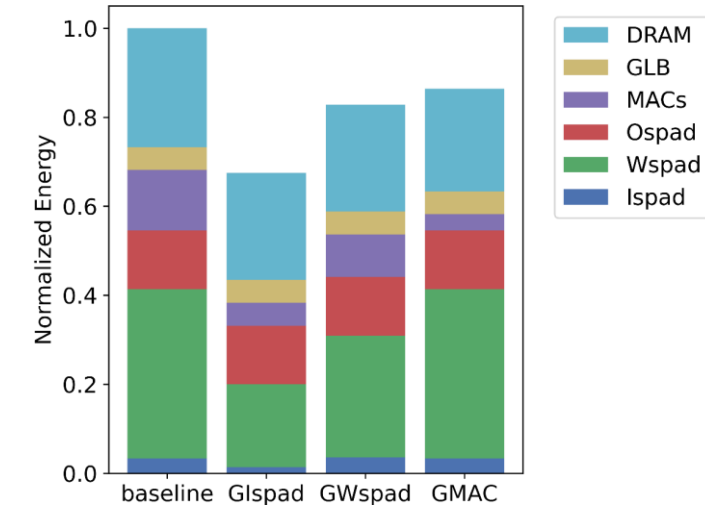
Gate Wspad PE Architecture



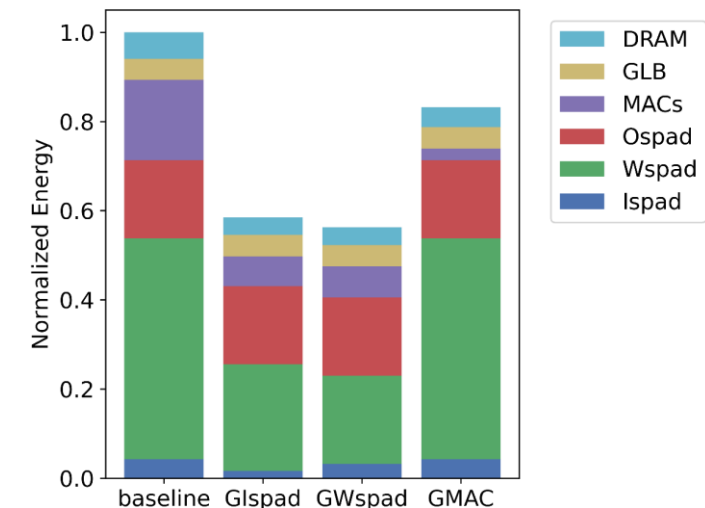
Gate Ispad PE Architecture

Larger extra metadata storage introduces more expansive access overhead (and area overhead)

Inception_3a_1x1



AlexNet Conv4



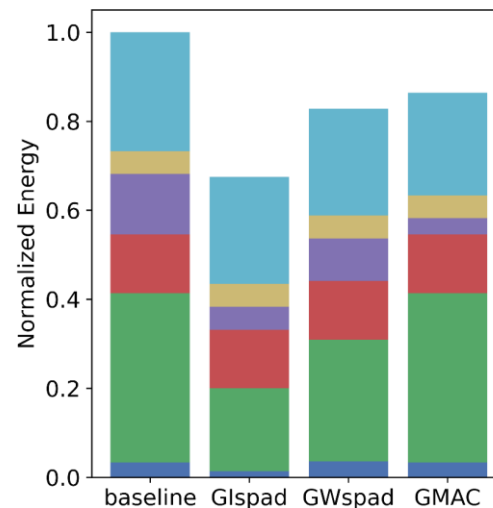
More Examples

Layer Densities

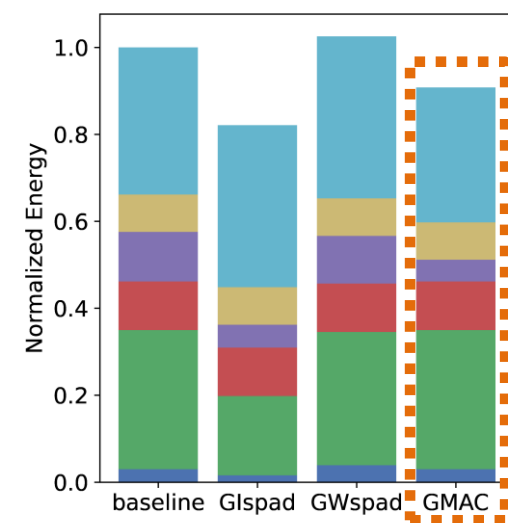
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Alexnet_conv4	0.39	0.43	0.37
Alexnet_conv5	0.43	0.16	0.37

Gate compute only could introduce better energy efficiency (and simpler hardware)

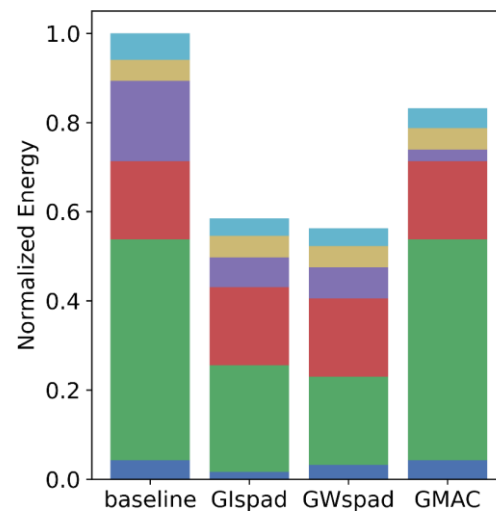
Inception_3a_1x1



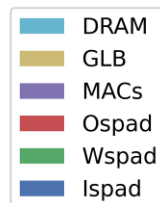
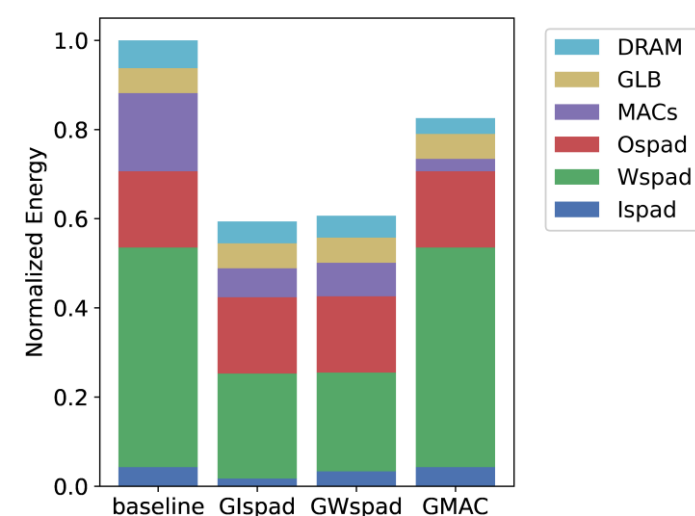
Inception_3a_pool_proj



AlexNet Conv4

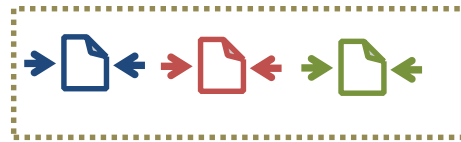
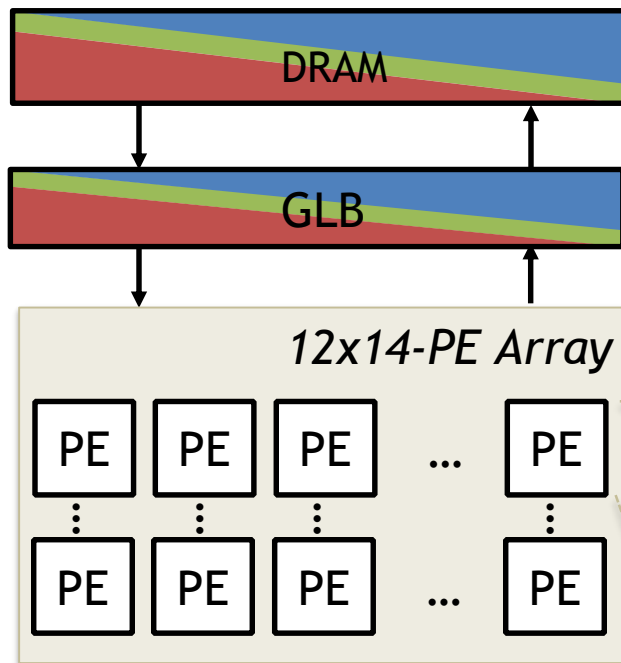


AlexNet Conv5



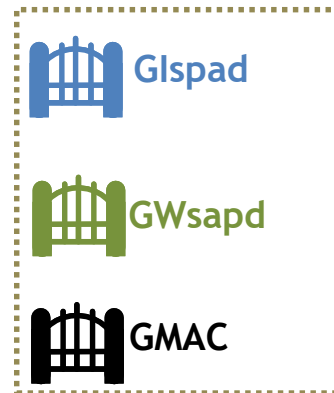
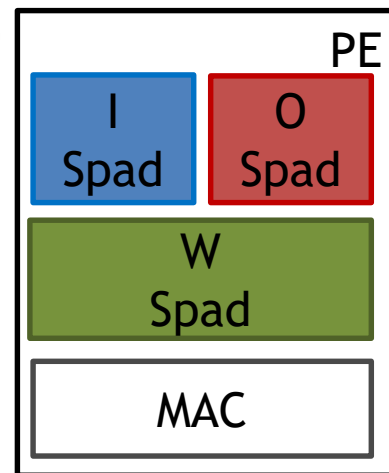
Explore different sparse optimization features

High-Level Architecture Setup



What are some important factors that define the impact of compressed data representation format?

- *Uncompressed traffic breakdown*
- *Tensor density*



What are some important factors that define the impact of gating on-chip?

- *Uncompressed traffic*
- *Tensor density*
- *Hardware attributes*

Sparse Tensor Accelerator Modeling Summary

- **Methodology**
 - Specifications
 - Mapping
 - Statistical workload density models
 - Sparse optimization features
 - Systematic analysis of the interactions between different specifications
 - Modularized modeling process that decouples dense traffic modeling and sparse optimization impact modeling
- **Timeloop V2 (a.k.a. Sparseloop) Infrastructure**
 - Implements the proposed methodology based on Timeloop V1
 - Modularized to allow data representation format and density model plug-ins
- **Validation and case studies**
 - Validation on Eyeriss V1 and SCNN
 - Exploration of various combinations of sparse optimization features

Sparse Tensor Accelerators: Abstraction and Modeling

Background Lecture Part 2

Joel Emer

Angshuman Parashar

Vivienne Sze

Po-An Tsai

Nellie Wu

ISCA Tutorial

June 2021

